KS5 "Full Coverage": Differentiation (Year 1)

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Determine the gradient of a curve at a particular point.

[Edexcel AS SAM P1 Q2]

The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point $P(5,6)$.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

Gradient = ...

(4 marks)

Question 2

Categorisation: Find the gradient at key points on the graph, e.g. roots.

[Edexcel C1 Jan 2012 Q8c] The curve C_1 has equation $y = x^2(x+2)$.

Find the gradient of C_1 at each point where C_1 meets the x -axis.

$m_1 =$ ....

$m_2 =$ ....

(2 marks)

Question 3

Categorisation: Find the gradient function where the expression involves roots.

[Edexcel A2 June 2018 P1 Q2ai]

A curve C has equation

$y = x^2 - 2x - 24\sqrt{x}, \quad x > 0$

Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ ....

(2 marks)

Question 4

Categorisation: Find the gradient function where the expression involves reciprocal terms.*[Edexcel FP1 June 2011 Q4a]*

$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0$

Use differentiation to find $f'(x)$.

$f'(x) =$ ....

(2 marks)

Question 5

Categorisation: Reciprocal root terms.

[OCR C1 June 2010 Q6]

Find the gradient of the curve $y = 2x + \frac{6}{\sqrt{x}}$ at the point where $x = 4$.

....

(5 marks)

Question 6

Categorisation: Differentiate where prior expansion is required.

[OCR C1 June 2017 Q3] It is given that $f(x) = (3 + x^2)(\sqrt{x} - 7x)$

Find $f'(x)$.

$f'(x) =$ ....

(5 marks)

Question 7

Categorisation: Differentiate by first principles.

[Edexcel AS June 2018 P1 Q10 Edited]

Prove, from first principles, that the derivative of x^3 is $3x^2$.

Given formula: $f'(x) = \ lim_{\ h \ to \ 0} \frac{f(x+h)-f(x)}{h}$

....

(4 marks)

Question 8

Categorisation: Differentiate by first principles when there are multiple terms.

[Edexcel A2 Specimen Papers P1 Q9] The curve C has equation

$$y = 2x^3 + 5$$

The curve C passes through the point $P(1,7)$. Use differentiation from first principles to find the value of the gradient of the tangent to C at P .

$\frac{dy}{dx} =$ ....

(5 marks)

Question 9

Categorisation: Determine the range of values for a quadratic function for which the function is increasing or decreasing.

[OCR C1 June 2013 Q9ii]

Find the set of values of x for which $2x^2 - x - 6$ is a decreasing function.

....

(3 marks)

Question 10

Categorisation: As above, but for a cubic function.

[Edexcel AS Specimen Papers P1 Q1b] A curve has equation

$$y = 2x^3 - 2x^2 - 2x + 8$$

Find the range of values of x for which y is increasing.

Write your answer in set notation.

....

(4 marks)

Question 11

Categorisation: Increasing/decreasing functions involving awkward powers.

[Edexcel C2 June 2010 Q3b Edited] $y = x^2 - k\sqrt{x}$, where k is a constant.

$$\frac{dy}{dx} = 2x - \frac{1}{2}kx^{-\frac{1}{2}}$$

Given that y is decreasing at $x = 4$, find the set of possible values of k .

....

(2 marks)

Question 12
Categorisation: Determine the stationary point(s) of a function.

[Edexcel C2 May 2013(R) Q1]

Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, x > 0$$

……………………

(6 marks)

Question 13
Categorisation: As above.

[Edexcel C2 June 2018 Q9a]

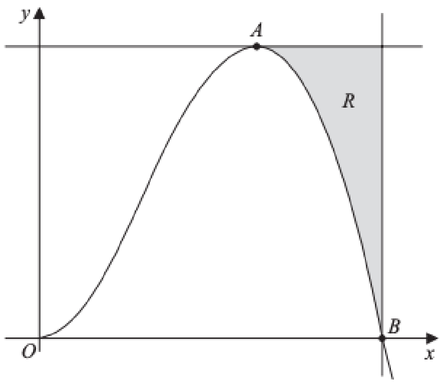


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$y = 7x^2(5 - 2\sqrt{x})$, $x \geq 0$

The curve has a turning point at the point A , where $x > 0$, as shown in Figure 3. Using calculus, find the coordinates of the point A .

……………………

(5 marks)

Question 14

Categorisation: Determine a turning point where the function involves fractional powers.

[Edexcel C2 Jan 2010 Q9a]

The curve *C* has equation $y = 12\sqrt{x} - x^{\frac{3}{2}} - 10\,, \quad x > 0\,.$

Use calculus to find the coordinates of the turning point on *C*.

....

(7 marks)

Question 15

Categorisation: Stationary points in a modelling context.

[Edexcel AS June 2018 P1 Q8a]

A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

Find, according to this model, the value of v that minimises the cost of the journey, and the minimum cost of the journey.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

$v =$ .... km h $^{-1}$

Minimum cost: £ ....

(6 marks)

Question 16

Categorisation: Stationary values in a geometric context.

[Edexcel AS SAM P1 Q16c Edited]

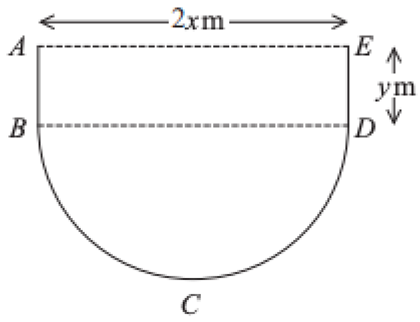


Figure 4

Figure 4 shows the plan view of the design for a swimming pool. The shape of this pool $ABCDEA$ consists of a rectangular section $ABDE$ joined to a semicircular section BCD as shown in Figure 4.

$AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m ^{2}, and the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

$P =$ .... m

(4 marks)

Question 17

Categorisation: Determine the second derivative.

[Edexcel A2 June 2018 P1 Q2aii] A curve C has equation

$y = x^2 - 2x - 24\sqrt{x}, \quad x > 0$

Find $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} =$ ....

(1 mark)

Question 18

Categorisation: Determine the nth term in an arithmetic sequence.

[Edexcel C2 Jan 2010 Q9c Edited]

The curve *C* has equation $y = 12\sqrt{x} - x^{\frac{3}{2}} - 10$, $x > 0$.

The curve C has a turning point at $(4,6)$.

Given that $\frac{d^2y}{dx^2} = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$

State the nature of the turning point.

....

(1 mark)

Question 19

Categorisation: Determine the equation of a tangent.

[Edexcel C1 Jan 2013 Q11b Edited]

The curve C has equation

$y = 2x - 8\sqrt{x} + 5$, $x \geq 0$

and hence $\frac{dy}{dx} = 2 - \frac{4}{\sqrt{x}}$.

The point P on C has x -coordinate equal to $\frac{1}{4}$. Find the equation of the tangent to C at the point P , giving your answer in the form $y = ax + b$, where a and b are constants.

....

Question 20

Categorisation: Determine the equation of the normal and use to make geometric calculations.

[Edexcel AS June 2018 P1 Q15 Edited]

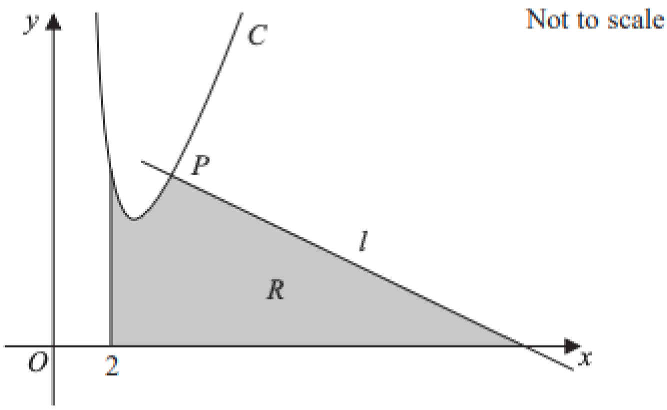


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$y = \frac{32}{x^2} + 3x - 8$, $\quad x > 0$

The point $P(4,6)$ lies on C.

The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Find the area of R.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

Area = ... unit ^{2}

(10 marks)

Question 21

Categorisation: Equation of the tangent where the gradient function is already given.

[Edexcel C1 May 2017 Q7a]

The curve *C* has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on *C*, find the equation of the tangent to *C* at P , giving your answer in the form $y = mx + c$, where m and c are constants.

....

(4 marks)

Question 22

Categorisation: Determine the point for which a particular gradient for the normal or tangent is achieved.

[Edexcel C1 May 2015 Q10b Edited]

A curve P has the gradient function

$$f(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2$$

where $x > 0$.

Point P lies on the curve. The normal to the curve at P is parallel to the line $2y + x = 0$.

Find the x -coordinate of P .

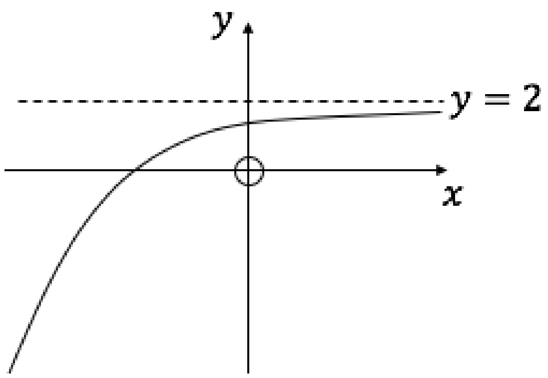
$x =$ ....

(5 marks)

Question 23

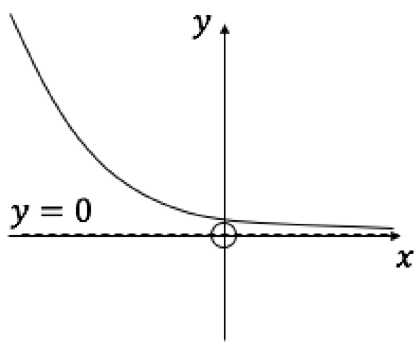
Categorisation: Draw/identify a sketch of a gradient function where the original function is given only in sketch form.

The curve with equation $y = f(x)$ is drawn below.

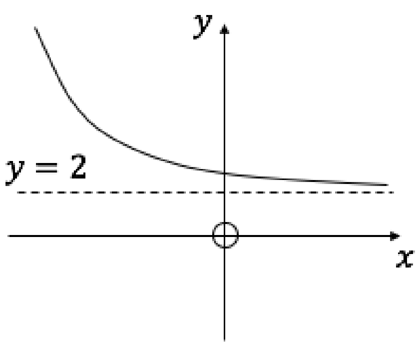


Which of the graph below is a sketch the gradient function of the curve $y = f(x)$?

Graph A



Graph B



[ ] A [ ] B

Question 24

Categorisation: Use the second derivative to determine the nature of a stationary point.

[Edexcel C2 May 2011 Q8c Edited]

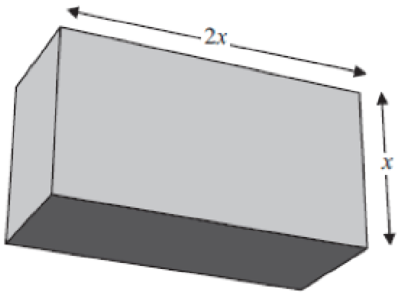


Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

The total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

The minimum value of L is 54 cm when $x = 3$.

Justify that the value of L is a minimum.

(2 marks)

Answers

Question 1

Gradient = 8

Attempt to differentiate	M1
$\frac{dy}{dx} = 4x - 12$	A1
Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = \ldots$	M1
$\Rightarrow \frac{dy}{dx} = 8$	A1ft

Question 2

(c)	At $x = -2$: $\frac{dy}{dx} = 3(-2)^2 + 4(-2) = 4$		M1	
	At $x = 0$: $\frac{dy}{dx} = 0$	(Both values correct)	A1	(2)

Question 3

$\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1

Question 4

$f(x) = x^2 + \frac{5}{2}x^{-1} - 3x - 1$		
$f'(x) = 2x - \frac{5}{2}x^{-2} - 3 \{+ 0\}$	At least two of the four terms differentiated correctly.	M1
	Correct differentiation. (Allow any correct unsimplified form)	A1
$\left\{f'(x) = 2x - \frac{5}{2x^2} - 3\right\}$		

Question 5

$\frac{13}{8}$

$y = 2x + 6x^{-\frac{1}{2}}$	**M1**		Attempt to differentiate
	A1		$kx^{-\frac{3}{2}}$
$\frac{dy}{dx} = 2 - 3x^{-\frac{3}{2}}$	**A1**		Completely correct expression (no +c)
When $x = 4$, gradient $= 2 - \frac{3}{\sqrt{4^3}}$	**M1**		Correct evaluation of either $4^{-\frac{3}{2}}$ or $4^{-\frac{1}{2}}$
$= \frac{13}{8}$	**A1**	**5** 5	

Question 6

$f(x) = \frac{3}{2}x^{-\frac{1}{2}} - 21 + \frac{5}{2}x^{\frac{3}{2}} - 21x^2$

$\sqrt{x} = x^{\frac{1}{2}}$ seen or implied	B1
$3x^{\frac{1}{2}} - 21x + x^{\frac{5}{2}} - 7x^3$	M1 A1
$\frac{3}{2}x^{-\frac{1}{2}} - 21 + \frac{5}{2}x^{\frac{3}{2}} - 21x^2$	M1 A1

Question 7

$3x^2h + 3xh^2 + h^3$

Considers $\frac{(x+h)^3 - x^3}{h}$	B1
Expands $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1
so gradient (of chord) $= \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$	A1
States as $h \to 0$, $3x^2 + 3xh + h^2 \to 3x^2$ so derivative $= 3x^2$ *	A1*

Question 8

$\frac{dy}{dx} = 6$

Gradient of chord $= \frac{(2(x+h)^3 - (2x^3 + 5)}{x + h - x}$	B1 M1	1.1b 2.1
$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	B1	1.1b
Gradient of chord $= \frac{(2(x^3 + 3x^2h + 3xh^2 + h^3) + 5) - (2x^3 + 5)}{1 + h - 1}$		
$= \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 5 - 2x^3 - 5}{1 + h - 1}$		
$= \frac{6x^2h + 6xh^2 + 2h^3}{h}$		
$= 6x^2 + 6xh + 2h^2$	A1	1.1b
$\frac{dy}{dx} = \lim_{h \to 0} \left(6x^2 + 6xh + 2h^2\right) = 6x^2$ and so at P, $\frac{dy}{dx} = 6(1)^2 = 6$	A1	2.2a

Question 9

$x < \frac{1}{4}$

$\frac{dy}{dx} = 4x - 1 = 0$	M1
Vertex when $x = \frac{1}{4}$	A1
$x < \frac{1}{4}$	A1 FT

Question 10

Attempts $6x^2-4x-2>0 \Rightarrow (6x+2)(x-1)>0$	M1	1.1b
$x=-\frac{1}{3},1$	A1	1.1b
Chooses outside region	M1	1.1b
$\left\{x: x<-\frac{1}{3}\right\} \cup \{x: x>1\}$	A1	2.5

Question 11

(b) Substituting $x=4$ into their $\frac{dy}{dx}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$)	M1
$8-\frac{k}{4}<0 \quad k>32 \quad (\text{or } 32<k)$ Correct inequality needed	A1

Question 12

$\frac{dy}{dx}=2-16x^{-3}$	M1 A1
$2-16x^{-3}=0$ so $x^{-3}=$ or $x^3=$, or $2-16x^{-3}=0$ so $x=2$	M1
$x=2$ only (after correct derivative)	A1
$y=2\times\text{"2"}+3+\frac{8}{\text{"}2^2\text{"}}$	M1
$=9$	A1

Question 13

$\frac{dy}{dx}=70x \;-35x^{\frac{3}{2}}$	M1A1
Put $\frac{dy}{dx}=0$ to give $70x \;-35x^{\frac{3}{2}}=0$ *so* $x^{\frac{1}{2}}=2$	M1
$x=4$	A1
$y=112$	A1

Question 14

(4,6)

$\left[y=12x^{\frac{1}{2}}-x^{\frac{3}{2}}-10\right]$	
$[y'=] \quad 6x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}}$	M1 A1
Puts their $\frac{6}{x^{\frac{1}{2}}}-\frac{3}{2}x^{\frac{1}{2}}=0$	M1
So $x=$ $,\frac{12}{3}=4$ (If $x=0$ appears also as solution then lose A1)	M1, A1
$x=4, \Rightarrow y=12\times2-4^{\frac{3}{2}}-10$, so $y=6$	dM1,A1

Question 15

$C=\frac{1500}{v}+\frac{2v}{11}+60 \Rightarrow \frac{dC}{dv}=-\frac{1500}{v^2}+\frac{2}{11}$	M1 A1
Sets $\frac{dC}{dv}=0 \Rightarrow v^2=8250$	M1
$\Rightarrow v=\sqrt{8250} \Rightarrow v=90.8\,(\text{km h}^{-1})$	A1
For substituting their $v=90.8$ in $C=\frac{1500}{v}+\frac{2v}{11}+60$	M1
Minimum cost =awrt (£) 93	A1 ft

Question 16

Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1} \rightarrow x^{-2}$	M1
$\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1
Sets $\frac{dP}{dx}=0$ and proceeds to $x=$	M1
Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8 M	A1

Question 17

$\frac{d^2y}{dx^2}=2+6x^{-\frac{3}{2}}$	B1ft

Question 18

[Since $x>0$] It is a maximum	B1

Question 19

$y=-6x+3$

Question 20

Area = 46 unit ^{2}

For the complete strategy of finding where the normal cuts the x-axis. Key points that must be seen are • Attempt at differentiation	M1

• Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis	
$y=\frac{32}{x^2}+3x-8 \Rightarrow \frac{dy}{dx}=-\frac{64}{x^3}+3$	M1 A1
For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{dy}{dx}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6=\text{"}-\frac{1}{2}\text{"}(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6=\text{"}-\frac{1}{2}\text{"}(x-4) \Rightarrow x=...$ or an attempt using just gradients $\text{"}-\frac{1}{2}\text{"}=\frac{6}{a-4} \Rightarrow a=...$	dM1
Normal cuts the x-axis at $x=16$	A1

For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{\text{"}16\text{"}} \text{"}\left(-\frac{1}{2}x+8\right)\text{"}dx$	M1
$\int \frac{32}{x^2}+3x-8\,dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1
Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1
Total area $=10+36=46$ *	A1*

Question 21

$y=-7x+20$

$f'(4)=30+\frac{6-5\times 4^2}{\sqrt{4}}$	Attempts to substitutes $x=4$ into $f'(x)=30+\frac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated $f'(x)$	M1
$f'(4)=-7$	Gradient = -7	A1
$y-(-8)=\text{"}-7\text{"}\times(x-4)$ or $y=\text{"}-7\text{"}x+c \Rightarrow -8=\text{"}-7\text{"}\times 4+c$ $\Rightarrow c=...$	Attempts an equation of a tangent using their numeric $f'(4)$ which has come from substituting $x=4$ into the given $f'(x)$ or their algebraically manipulated $f'(x)$ and $(4,-8)$ with the 4 and -8 correctly placed. If using $y=mx+c$, must reach as far as $c=...$	M1
$y=-7x+20$	Cao. Allow $y=20-7x$ and allow the "$y=$" to become "detached" but it must be present at some stage. E.g. $y=...$ $=-7x+20$	A1

Question 22

$x = 1.5$

Gradient of normal is $-\frac{1}{2} \Rightarrow$ Gradient of tangent = +2	M1: Gradient of $2y + x = 0$ is $\pm\frac{1}{2}(m) \Rightarrow \frac{dy}{dx} = -\frac{1}{\pm\frac{1}{2}}$ A1: Gradient of tangent = +2 (May be implied)	M1A1
The A1 may be implied by $\frac{-1}{\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2} = -\frac{1}{2}$		
$\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2 = 2 \Rightarrow \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} = 0$	Sets the given $f'(x)$ **or their** $f'(x)$ = their **changed** m and **not** their m where m has come from $2y + x = 0$	M1
$\times 4\sqrt{x} \Rightarrow 6x - 9 = 0 \Rightarrow x = ..$	$\times 4\sqrt{x}$ or equivalent **correct algebraic** processing (**allow sign/arithmetic errors only**) and attempt to solve to obtain a value for x. If $f'(x) \neq 2$ they need to be solving a three term quadratic in $\sqrt{x}$ correctly and square to obtain a value for x. **Must be using the given** $f'(x)$ **for this mark.**	M1
$x = 1.5$	$x = \frac{3}{2}$ (1.5) Accept equivalents e.g . $x = \frac{9}{6}$ **If any 'extra' values are not rejected, score A0.**	A1

Question 23

A

Question 24

When $x = 3$, $\frac{d^2L}{dx^2} = 12$

$\{\text{For } x = 3\}$, $\frac{d^2L}{dx^2} = \frac{972}{x^4} > 0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
	$\frac{972}{x^4}$ and > 0 and conclusion.	A1