BRONZE

1. The rectangle R has vertices at the points $(0, 0)$, $(1, 0)$, $(1, 2)$ and $(0, 2)$.

    (a)   Find the coordinates of the vertices of the image of R under the transformation given by the matrix $\mathbf{A} = \begin{pmatrix} a & 5 \\ -1 & 1 \end{pmatrix}$, where a is a constant.

                                         (3)

    (b)   Find det $\mathbf{A}$, giving your answer in terms of a.

                                         (1)

Given that the area of the image of R is 18,

    (c)   find the value of a.

                                       (3)
                        (Total 7 marks)

SILVER

2. The matrix $\mathbf{R}$ is given by $\mathbf{R} = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}$

    (a)   Find $\mathbf{R}^2$.

                                         (2)

    (b)   Describe the geometrical transformation represented by $\mathbf{R}^2$.

                                         (2)

    (c)   Describe the geometrical transformation represented by $\mathbf{R}$.

                                         (1)
                        (Total 5 marks)

GOLD

3.
$$\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}$$

(a)   Describe fully the transformations described by each of the matrices **A**, **B** and **C**.

(4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(b)   Find **D**.

(2)

(c)   Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$.

(1)

The triangle *ORS* has vertices at the points with coordinates (0, 0), (–15, 15) and (4, 21).
This triangle is transformed onto the triangle *OR′S′* by the transformation described by **E**.

(d)   Find the coordinates of the vertices of triangle *OR′S′*.

(4)

(e)   Find the area of triangle *OR′S′* and deduce the area of triangle *ORS*.

(3)
(Total 14 marks)