OCR Maths FP1

Topic Questions from Papers

Complex Numbers

Answers

1	(i) $22 - 2i$	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$	B1		Correct conjugate seen or implied
	$5 - 14i$	B1B1	3	Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1		Attempt to use w^*
		A1	2	Obtain correct answer in any form
			7	

(Q3, June 2005)

2	$x^2 - y^2 = 21$ and $xy = -10$	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and $21 - 20i$
		A1A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)5$ or $y = (\pm)2$
	$\pm(5 - 2i)$	A1	6	Obtain correct answers as complex numbers
			6	

(Q4, June 2005)

3	(i) Circle	B1		Sketch(s) showing correct features, each mark independent
	Centre (0, 2)	B1		
	Radius 2	B1		
	Straight line	B1		
	Through origin with positive slope	B1	5	
	(ii) 0 or 0 +0i and 2 + 2i	B1ftB1ft	2	Obtain intersections as complex numbers
			7	

(Q6, June 2005)

4	(i) $2 + 16i - i - 8i^2$	M1		Attempt to multiply correctly
	$10 + 15i$	A1	2	Obtain correct answer
	(ii)	M1		Multiply numerator & denominator by conjugate
		A1		Obtain denominator 5
	$\frac{1}{5}(10 + 15i)$ or $2 + 3i$	A1ft	3	Their part (i) or $10 + 15i$ derived again / 5
			5	

(Q1, Jan 2006)

5	(a) (i) $\sqrt{13}$	B1	1	Obtain correct answer, decimals OK
	(ii)			
		M1		Using $\tan^{-1} b/a$, or equivalent trig allow + or -
	- 0.59	A1		Obtain 0.59
		A1	3	Obtain correct answer
	(b)			
		M1		Express LHS in Cartesian form & equate real and imaginary parts
	1 – 2i	A1A1		Obtain $x = 1$ and $y = -2$
		A1	4	Correct answer written as a complex number
	(c)	B1		Sketch of vertical straight line
		B1	2	Through (- 0.5, 0)
			10	

(Q7, Jan 2006)

6	(i) 2 + 3i	B1	1	Conjugate seen

(Q3, June 2006)

7	(i) -7i	B1		Real part correct
		B1	2	Imaginary part correct
	(ii) 2 + 3i	B1		iz stated or implied or $i^2 = -1$ seen
		B1		Real part correct
	-5 + 12i	B1	3	Imaginary part correct
	(iii) $\frac{1}{5}(4 - 7i)$ or equivalent	M1		Multiply by conjugate
		A1		Real part correct
		A1	3	Imaginary part correct
			8	**N.B. Working must be shown**

(Q5, June 2006)

8	(i) Circle , Centre *O* radius 2	B1 B1		Sketch showing correct features
	One straight line	B1		
	Through *O* with +ve slope	B1		
	In 1st quadrant only	B1	5	
	(ii) $1 + i\sqrt{3}$	M1		Attempt to find intersections by trig, solving equations or from graph
		A1	2	Correct answer stated as complex number
			7	

(Q6, June 2006)

9		M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and 15 +8i
	$x^2 - y^2 = 15$ and $xy = 4$	A1 A1		
		M1		Obtain each result
		DM1		Eliminate to obtain a quadratic in x^2 or y^2
	$\pm(4 + i)$	A1	6	
			6	Solve to obtain $x = (\pm)4$, or $y = (\pm)1$
				Obtain only correct two answers as complex numbers

(Q2, Jan 2007)

10	(i)	B1		Circle
		B1		Centre (1, -1)
		B1	3	Passing through (0, 0)
	(ii)	B1		Sketch a concentric circle
		B1		Inside (i) and touching axes
		B1	3	Shade between the circles

(Q4, Jan 2007)

11	(i)	B1	1	Show given answer correctly
	(ii)	M1		Attempt to solve quadratic equation or substitute $x + iy$ and equate real and imaginary parts
	$-1 \pm i\sqrt{3}$	A1		Obtain answers as complex numbers
		A1	3	Obtain correct answers, simplified
	(iii)	B1		Correct root on x axis, co-ords. shown
		B1		Other roots in 2nd and 3rd quadrants
		B1		
			3	Correct lengths and angles or co-ordinates or complex numbers shown
			7	

(Q5, Jan 2007)

12	*EITHER* $a = 2$ $b = 2\sqrt{3}$, *OR* $a = 2 \quad b = 2\sqrt{3}$	M1 A1 M1 A1 M1 M1 A1 A1	 4 **4**	Use trig to find an expression for a (or b) Obtain correct answer Attempt to find other value Obtain correct answer a.e.f. (Allow 3.46) State 2 equations for a and b Attempt to solve these equations Obtain correct answers a.e.f. SR ± scores A1 only

(Q1, June 2007)

13	(i) Circle, centre (3, 0), *y*-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only (ii) Inside circle, below line, above *x*-axis	B1B1 B1 B1 B1 B1 B2ft	 6 2 **8**	Sketch showing correct features N.B. treat 2 diagrams asa MR Sketch showing correct region SR: B1ft for any 2 correct features

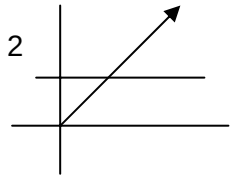
(Q8, June 2007)

14	(i) $x^2 - y^2 = 16$ and $xy = 15$ $\pm(5 + 3i)$ (ii) $z = 1 \pm \sqrt{16 + 30i}$ 6 + 3i, -4 - 3i	M1 A1A1 M1 M1 A1 M1* A1 *M1dep A1 A1ft	 6 5 **11**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and 16+30i Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,3$ Obtain correct answers as complex numbers Use quadratic formula or complete the square Simplify to this stage Use answers from (i) Obtain correct answers

(Q10, June 2007)

15	(i) $z^* = 3 + 4i$	B1		Conjugate seen or implied
	$21 + 12i$	B1	2	Obtain correct answer
	(ii) $3 - 5i$	B1		Correct $z - i$ or expansion of $(z - I)^2$ seen
		B1ft		Real part correct
	$-16 - 30i$	B1ft	3	Imaginary part correct
	(iii)	M1		Multiply by conjugate
	$\frac{9}{25} + \frac{12}{25}i$	A1		Numerator correct
		A1	3	Denominator correct
			8	

(Q4, Jan 2008)

16	(i) 	B1		Horizontal straight line in 2 quadrants
		B1		Through (0, 2)
		B1		Straight line
		B1		Through O with positive slope
		B1	5	In 1st quadrant only
	(ii)	B1		State or obtain algebraically that $y = 2$
	$2\sqrt{3} + 2i$	M1		Use suitable trigonometry
		A1	3	Obtain correct answer a.e.f. decimals OK must be a complex number
			8	

(Q6, Jan 2008)

17 (i) 5 — **B1** Correct modulus

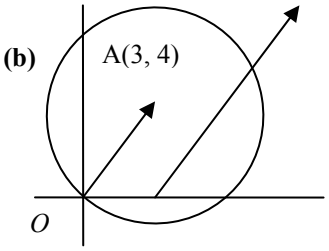
0.927 or 53.1° — **B1** Correct argument, any equivalent form

2

(ii)(a) — **B1** Circle centre A (3, 4)

B1 Through O, allow if centre is (4, 3)

2

(b) 

B1 Half line with +ve slope

B1 Starting at (3, 0)

B1 Parallel to OA, (implied by correct arg shown)

3

(Q2, June 2008)

18 (i) — **M1** Attempt to equate real and imaginary parts of $(x + iy)^2$ and $5 + 12i$

$x^2 - y^2 = 5$ and $xy = 6$ — **A1** Obtain both results

M1 Eliminate to obtain a quadratic in x^2 or y^2

$\pm(3 + 2i)$ — **M1** Solve a 3 term quadratic & obtain x or y

A1 Obtain correct answers as complex nos.

5

(ii) $5 - 12i$ — **B1B1** Correct real and imaginary parts

2

(iii) — **M1** Attempt to solve a quadratic equation

$x^2 = 5 \pm 12i$ — **A1** Obtain correct answers

$x = \pm(3 \pm 2i)$ — **A1A1** Each pair of correct answers a.e.f.

4

(Q9, June 2008)

19	$\frac{7}{26} + \frac{17}{26}$ i.	M1 A1 A1 A1	 4 **4**	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator

(Q1, Jan 2009)

20	**(i)** $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1		Eliminate to obtain quadratic in x^2 or y^2
		M1		Solve to obtain x (or y) values
	$x = \pm\frac{\sqrt{10}}{2}, y = \pm\frac{\sqrt{2}}{2}$	A1		Correct values for both x & y obtained a.e.f.
	$\pm(\frac{\sqrt{10}}{2} + \text{i}\frac{\sqrt{2}}{2})$	A1	6	Correct answers as complex numbers
	(ii) $z^2 = 2 \pm \text{i}\sqrt{5}$	M1		Solve quadratic in z^2
		A1		Obtain correct answers
	$z = \pm(\frac{\sqrt{10}}{2} \pm \text{i}\frac{\sqrt{2}}{2})$	M1		Use results of (i)
		A1ft	4	Obtain correct answers, ft must include root from conjugate
	(iii)	B1ft	1	Sketch showing roots correctly
	(iv)	B1 B1ft		Sketch of straight line, $\perp$ to α
		B1ft	3 **14**	Bisector

(Q10, Jan 2009)

21	(i) 11 – 29i	B1 B1	2	Correct real and imaginary parts
	(ii) 1 + 41i	B1 B1	2 **4**	Correct real and imaginary parts

(Q3, June 2009)

22	(i) $3\sqrt{2}, -\frac{\pi}{4}$ or –45° AEF	B1 B1	2	State correct answers
	(ii)(a)	B1B1 B1 ft	3	Circle, centre (3, -3), through *O* ft for (±3,±3) only
	(ii)(b)	B1 B1 B1	3	Straight line with +ve slope, through (3, -3) or their centre Half line only starting at centre
	(iii)	B1ft B1ft B1ft	3	Area above horizontal through *a*, below (ii) (b) Outside circle
			11	

(Q6, June 2009)

23	$x - iy$	B1		Conjugate known
		M1		Equate real and imaginary parts
	$x + 2y = 12 \quad 2x + y = 9$	A1		Obtain both equations, OK with factor of i
		M1		Solve pair of equations
	$z = 2 + 5i$	A1	**5**	Obtain correct answer as a complex number **S.C.** Solving $z + 2iz = 12 + 9i$ can get max 4/5, not first B1
		5		

(Q3, Jan 2010)

24 (i)		M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ & $5 - 12i$
	$x^2 - y^2 = 5$ and $xy = -6$	A1		Obtain both results, a.e.f
		M1		Obtain quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
	$\pm(3 - 2i)$	A1	**5**	Obtain correct answers as complex nos
	(ii)			B1ft Circle with centre at their square root
		B1		Circle passing through origin
		B1ft		2nd circle centre correct relative to 1st
		B1	**4**	Circle passing through origin
		9		

(Q8, Jan 2010)

25	**(i)** 5 + 12i	B1B1		Correct real and imaginary parts
	13	B1ft		Correct modulus
	67.4^o or 1.18	B1ft	**4**	Correct argument
	(ii)	M1		Multiply by conjugate
		A1		Obtain correct numerator
	$-\frac{11}{85} - \frac{27}{85}$ i	A1	**3**	Obtain correct denominator
		7		

(Q4, June 2010)

26 (i)	**(a)**	B1B1	**2**	Circle centre (3, –4), through origin
	(b)	B1B1	**2**	Vertical line, clearly $x = 3$
(ii)		B1ft		Inside their circle
		B1ft	**2**	And to right of their line, if vertical
			6	

(Q6, June 2010)

27	**(i)**	M1		Attempt to equate real and imaginary parts
	$x^2 - y^2 = 3 \quad xy = 2$	A1		Obtain both results
		M1		Eliminate to obtain quadratic in x^2 or y^2
		M1		Solve to obtain x or y value
	$z = 2 + \mathrm{i}$	A1	**5**	Obtain correct answer as a complex no.
	(ii)	B1	**1**	Obtain **given** answer correctly
	(iii)	M1		Attempt to solve quadratic equation
	$w^3 = 2 \pm 11\,\mathrm{i}$	A1		Obtain correct answers
		M1		Choose negative sign
		M1		Relate required value to conjugate of (i)
	$w = 2 - \mathrm{i}$	A1	**5**	Obtain correct answer
			11	

(Q10, June 2010)

28 (i)	$-12 + 13\mathrm{i}$	B1B1	**2**	Real and imaginary parts correct
(ii)		B1		z^* seen
		M1		Multiply by w^*
	$\frac{27}{37} - \frac{14}{37}\,\mathrm{i}$	A1		Obtain correct real part or numerator
		A1	**4**	Obtain correct imaginary part or denom. Sufficient working must be shown
			6	

(Q2, Jan 2011)

29 (i)	**(a)**	B1*		Vertical line
		depB1	**2**	Clearly through (4, 0)
	(b)	B1		Sloping line with +ve slope
		B1		Through (0, -2)
		B1ft	**3**	Half line starting on *y*-axis 45^o shown convincingly
(ii)		B1ft		Shaded to left of their **(i) (a)**
		B1ft		Shaded below their **(i) (b)** must be +ve slope
		B1ft	**3**	Shaded above horizontal through their (0, -2)
				NB These 3 marks are independent, but 3/3 only for fully correct answer.
			8	

(Q6, Jan 2011)

30 (i)	$\lvert a\rvert = 2$	B1		Correct modulus
	$\arg a = 60°, \frac{\pi}{3}$, 1.05	B1	**2**	Correct argument
(ii)		B1		Circle
		B1		Centre $(1, \sqrt{3})$
		B1		Through origin, centre $(\pm 1, \pm\sqrt{3})$ and another y intercept
		B1		Vertical line
		B1*		Through *a* or their centre, with +ve gradient
		DB1		Correct half line
			6	
			8	

(Q5, June 2011)

31 (i)	16 + 30i	B1	**1**	State correct value
(ii)		M1		Use *a* = – (sum of roots)
	a = –32	A1		Obtain correct answer
		M1		Use *b* = product of roots
	b = 1156	A1	**4**	Obtain correct answer
		M1		Substitute, expand and equate imag. parts
		A1		Obtain **a** = -32
		M1		Equate real parts
		A1		Obtain b = 1156
(iii)		M1		Attempt to equate real and imaginary parts of $(p+iq)^2$ & 16 – 30i or root from (ii)
	$p^2 - q^2 = 16$ and $pq = -15$	A1		Obtain both results cao
		M1		Obtain quadratic in p^2 or q^2
		M1		Solve to obtain $p = (\pm)5$ or $q = (\pm)3$
		A1		Obtain 2 correct answers as complex nos
		M1		Attempt at all 4 roots
	± (5 ± 3i)	A1	**7**	State other two roots as complex nos
			12	

(Q9, June 2011)

32			$a^2 + 5^2 = 13^2$ $a = 12$ $\tan^{-1}\frac{5}{a}$ 0.395 or 22.6° or 0.126π	M1 A1 M1 A1FT **[4]**	Use formula for modulus Obtain correct answer Use formula for argument Obtain correct answer allow 0.39	

(Q1, Jan 2012)

33			$x^2 - y^2 = 3$ and $xy = 3\sqrt{2}$ $x^4 - 3x^2 - 18 = 0$ or $y^4 + 3y^2 - 18 = 0$ $x = \pm\sqrt{6}$ or $y = \pm\sqrt{3}$ $\pm(\sqrt{6} + i\sqrt{3})$	M1 A1 M1 M1 A1 A1 **[6]**	Attempt to equate real and imaginary parts Obtain both results Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x or y value Both values correct Correct answers as complex numbers	

(Q3, Jan 2012)

34				B1 B1 B1 B1 B1 B1 **[6]**	Circle Centre $(\sqrt{3},1)$ Passing through O and crosses y-axis again Line, with correct slope shown $\frac{1}{2}$ line starting at O Completely correct diagram for both loci	Ignore shading

(Q6, Jan 2012)

35	(i)		21 +11i	B1 B1 **[2]**	Real part correct Imaginary part correct	
	(ii)		26 – 29i $\frac{26}{41} - \frac{29}{41}i$	M1 A1 A1 **[3]**	Multiply by conjugate of denominator or find a pair of simultaneous equations Obtain correct numerator or real part Obtain correct denominator or imaginary part	

(Q1, June 2012)

36	(i)			B1B1 B1ft B1ft B1B1 **[6]**	Circle, centre (3 , 4) Touching x-axis, ft for $(3, -4)$ etc as centre Crossing y-axis twice Horizontal line, y intercept 4	
	(ii)		–1 +4i 7 + 4i	B1B1 **[2]**	State correct answers	
	(iii)			B1ft B1 **[2]**	Inside circle or above line Completely correct diagram	

(Q7, June 2012)

37	**(i)**		$\|z\| = \sqrt{5}$	B1	Allow 2.2
			$\arg z = -26.6^\circ$ or -0.464	B1 **[2]**	Allow -27^o or -0.46(3)
	(ii)			B1	$z^* = 2 + i$ stated or used
				M1	Obtain two equations from real and imaginary parts
			$a + b = 2, b - a = -8$	A1	Obtain correct equations
				M1	Attempt to solve 2 linear equations
			$a = 5,\ b = -3$	A1 **[5]**	Obtain correct answers

(Q3, Jan 2013)

38	**(i)**	**(a)**		B1 B1 **[2]**	Circle Centre *O* and radius 2
	(i)	**(b)**		B1 B1 B1 **[3]**	Horizontal line (3, 1) on their line ½ line to left i.e. horizontal
	(ii)			B1 B1 **[2]**	Shade only inside their circle or above their horizontal line Completely correct diagram

(Q7, Jan 2013)

39				M1	Use correct trig expression
			$\sqrt{3}$	A1	Obtain correct answer
				M1	Correct expression for modulus
			$2\sqrt{3}$	A1FT	Obtain correct answer aef
			$3 - \sqrt{3}i$	B1FT	Correct conjugate seen or implied
			$-\sqrt{3}\,i$	B1FT **[6]**	Correct answer

(Q1, June 2013)

40			$x^2 - y^2 = 11$ and $xy = 6\sqrt{5}$	M1	Attempt to equate real and imaginary parts of $(x + iy)^2$ and $11 + 12\sqrt{5}$
				A1	Obtain both results cao
				M1*	Obtain a quadratic in x^2 or y^2
			$\pm(2\sqrt{5} + 3i)$	DM1	Solve a 3 term quadratic to obtain a value for x or y
				A1	Obtain 1 correct answer as complex number
				A1 **[6]**	Obtain only the other correct answer

(Q3, June 2013)

41	**(i)**			M1	Use arg (z – a) = θ in equation for *l* condone missing brackets
			$\arg(z - 3i) = \frac{1}{4}\pi$	A1	Obtain correct answer
				M1	Use $\|z - a\| = k$ in equation for *C*, *k* must be real
			$\|z - 3i\| = 3$	A1 **[4]**	Obtain correct answer
	(ii)		$\|z - 3i\| \le 3$ or e.g. $x^2 + (y-3)^2 \le 9$	B1	Obtain correct inequality, or answer consistent with sensible **(i)**
			$\frac{1}{4}\pi \le \arg(z - 3i) \le \frac{1}{2}\pi$ or $y \ge x + 3,\ x \ge 0$	B1 B1 **[3]**	Each correct single inequality, or answer consistent with sensible **(i)** **SC if < used consistently, but otherwise all correct, B2**

(Q6, June 2013)