FP1 – Series Questions ANSWERS (42 marks)

Jan 2013

1.	$\sum_{r=1}^{n} 3(4r^2 - 4r + 1) = 12\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + \sum_{r=1}^{n} 3$ $= \frac{12}{6}n(n+1)(2n+1) - \frac{12}{2}n(n+1), \quad +3n$ $= n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$ $= n\left[4n^2 - 1\right] = n(2n+1)(2n-1)$	M1 A1, B1 M1 A1 cso **[5]**
Notes:	Induction is not acceptable here First M for expanding given expression to give a 3 term quadratic and attempt to substitute. First A for first two terms correct or equivalent. B for $+3n$ appearing Second M for factorising by n Final A for completely correct solution	

June 2012

4. (a)	$\sum_{r=1}^{n}\left(r^3+6r-3\right)$		
	$=\underline{\frac{1}{4}n^2(n+1)^2+6.\frac{1}{2}n(n+1)}-3n$	M1; An attempt to use at least one of the standard formulae correctly in summing at least 2 terms of r^3+6r-3 A1: Correct underlined expression. B1: $-3\to -3n$	M1A1B1
	$=\frac{1}{4}n^2(n+1)^2+3n^2+3n-3n$		
	If any marks have been lost, no further marks are available in part (a)		
	$=\frac{1}{4}n^2(n+1)^2+3n^2$ $=\frac{1}{4}n^2\left((n+1)^2+12\right)$	Cancels out the $3n$ and attempts to factorise out at least $\frac{1}{4}n$.	dM1
	$=\frac{1}{4}n^2\left(n^2+2n+13\right)$ **(AG)**	Correct answer **with no errors seen.**	A1 *
	Provided the first 3 marks are scored, allow the next two marks for correctly showing the algebraic equivalence. E.g. showing that both $\frac{1}{4}n^2(n+1)^2+6.\frac{1}{2}n(n+1)-3n$ and $\frac{1}{4}n^2\left(n^2+2n+13\right)=\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{13}{4}n^2$		
	There are no marks for proof by induction but apply the scheme if necessary.		
			[5]
(b)	$S_n=\sum_{r=16}^{30}\left(r^3+6r-3\right)=S_{30}-S_{15}$		
	$=\frac{1}{4}(30)^2\left(30^2+2(30)+13\right)-\frac{1}{4}(15)^2\left(15^2+2(15)+13\right)$	<u>Use</u> of $S_{30}-S_{15}$ or $S_{30}-S_{16}$	M1
	NB They must be using $S_n=\frac{1}{4}n^2\left(n^2+2n+13\right)$ **not** $S_n=n^3+6n-3$		
	$=218925-15075$		
	$=203850$	203850	**A1 cao**
	NB $S_{30}-S_{16}=218925-19264=199661$ (Scores M1 A0)		

7.	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
(a)	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= 4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1) + n$	First two terms correct.	A1
		$+ n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Attempt to factorise out $\frac{1}{3}n$	M1
		Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$ $= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$ $= \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
	Note that there are no marks for proof by induction.		
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	Note that (b) says **hence** so they have to be using the result from (a)		
	$= n(36n^2-1) - \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2 - 1)$	$\frac{2}{3}n(52n^2-1)$	A1
	$\{a = 52,\ b = -1\}$		(4)

Jan 2011

5	$\sum_{r=1}^{n} r(r+1)(r+5)$		
(a)	$= \sum_{r=1}^{n} r^3 + 6r^2 + 5r$	Multiplying out brackets and an attempt to use at least one of the standard formulae correctly.	M1
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{6}n(n+1)(2n+1) + 5.\frac{1}{2}n(n+1)}$	Correct expression.	A1
	$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$		
	$= \frac{1}{4}n(n+1)\left(n(n+1) + 4(2n+1) + 10\right)$	Factorising out at least $n(n+1)$	dM1
	$= \frac{1}{4}n(n+1)\left(n^2 + n + 8n + 4 + 10\right)$		
	$= \frac{1}{4}n(n+1)\left(n^2 + 9n + 14\right)$	Correct 3 term quadratic factor	A1
	$= \frac{1}{4}n(n+1)(n+2)(n+7)$ *	Correct proof. No errors seen.	A1 (5)
(b)	$S_n = \sum_{r=20}^{50} r(r+1)(r+5)$		
	$= S_{50} - S_{19}$		
	$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$	Use of $S_{50} - S_{19}$	M1
	$= 1889550 - 51870$		
	$= 1837680$	1837680 Correct answer only 2/2	A1
			(2) **[7]**

June 2009

Q3 (a)	$r(r+2)(r+4) = r^3 + 6r^2 + 8r$, so use $\sum r^3 + 6\sum r^2 + 8\sum r$ $= \frac{1}{4}n^2(n+1)^2 + 6\left(\frac{1}{6}n(n+1)(2n+1)\right) + 8\left(\frac{1}{2}n(n+1)\right)$ $= \frac{1}{4}n(n+1)\{n(n+1) + 4(2n+1) + 16\}$ $= \frac{1}{4}n(n+1)\{n^2 + 9n + 20\} = \frac{1}{4}n(n+1)(n+4)(n+5)$ (*)	1M1 1A1 2M1 2A1 3A1	 (5)
(b)	$\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{20}$ $= \frac{1}{4}(30\times31\times34\times35) - \frac{1}{4}(20\times21\times24\times25) = 213675$	1M1 1A1	 (2) [7]
(a)	Alternative (induction): $\frac{1}{4}k(k+1)(k+4)(k+5) + (k+1)(k+3)(k+5)$ 1M1 (Adding on (k+1)th term $= \frac{1}{4}(k+1)(k+5)\left(k^2 + 4k + 4k + 12\right)$ 2M1 Quadratic factor seen $= \frac{1}{4}(k+1)(k+2)(k+5)(k+6)$ 1A1 cso Check for k = 1: Term = 15, Sum = $\frac{60}{4} = 15$ 1B1 cao Induction argument + conclusion 2A1 cao		
(a) (b)	Q3 Notes 1M1 Expand in terms of $\sum r^3, \sum r^2, \sum r$ 1A1 Correct substitution in correct expansion. 2M1 Factorisation, 3 term quadratic factor seen 2A1 a correct quadratic factor 3A1 cso 1M1 allowed for $\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{19}$ or $\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{21}$ but must be used. 1A1 cao		

Jan 2009

1 (a)	$\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$	M1, A1	
	Simplifying this expression	M1	
	$= \frac{1}{3}n(n^2-4)$ (✱)	A1 cso	(4)
(b)	$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) = \frac{1}{3}\times 20\times(20^2-4) - \frac{1}{3}\times 9\times(9^2-4)$	M1	
	$= 2409$	A1	(2)
Alt (b)	$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) =$		
	$\left(\frac{1}{6}\times 20\times 21\times 41 - \frac{1}{2}\times 20\times 21 - 20\right) - \left(\frac{1}{6}\times 9\times 10\times 19 - \frac{1}{2}\times 9\times 10 - 9\right)$ M1		
	$= 2409$ A1		[6]
Notes	(a) 1st M: Separating, substituting set results, at least two correct. 2nd M: Either "eliminate" brackets totally or factor x [......] where any product of brackets inside [....] has been reduced to a single bracket 2nd A: ANSWER GIVEN. No wrong working seen; must have been an intermediate step, e.g. $\frac{1}{6}n(2n^2+3n+1-3n-3-6)$. (b) M: Must be $\sum_{r=1}^{20}(....) - \sum_{r=1}^{9}(....)$ applied. If list terms and add, allow M1 if **11 terms** with **at most two wrong**: [89, 109, 131, 155, 181, 209, 239, 271, 305, 341, 379]		