CM

A Level Maths Question Countdown
1 day until the 1ˢᵗ exam

Information
• Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self	

Pure questions – standard

1 (a) Given that

$$x = 1 - \ln(y^2), \qquad y \in \mathbb{R}, \ y > 0$$

find $\dfrac{dy}{dx}$ in terms of x, simplifying your answer.

(b) Given that

$$x = \frac{1}{\cos 2y}, \qquad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

show that $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{1-x^2}}$.

2 (a) The sequence of numbers $a_1, a_2, a_3, \ldots$, is defined such that
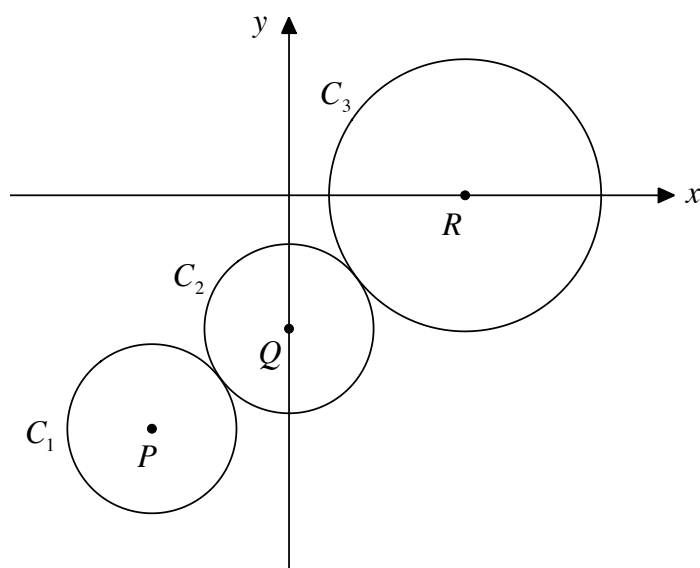
$$a_1 = k, \qquad k \neq 1, \ 2$$

$$a_{n+1} = \frac{3 - a_n}{2 - a_n}, \qquad n \in \mathbb{N}$$

Show that $a_1 = a_4$.

(b) Given that $k = 5$, find $\displaystyle\sum_{r=9}^{100} a_r$.

Pure questions – problems

3

Three circles C_1, C_2 and C_3 touch externally, as shown in the diagram above. The points P, Q and R are the centers of C_1, C_2 and C_3 respectively. The points P, Q and R lie on a straight line and Q lies on the y axis.

The equation of C_1 is $(x + 4)^2 + (y + 5)^2 = 5$.
The equation of C_3 is $(x - 6)^2 + y^2 = 20$.

(a) (i) Find the exact distance between the points P and R. Simplify your answer.
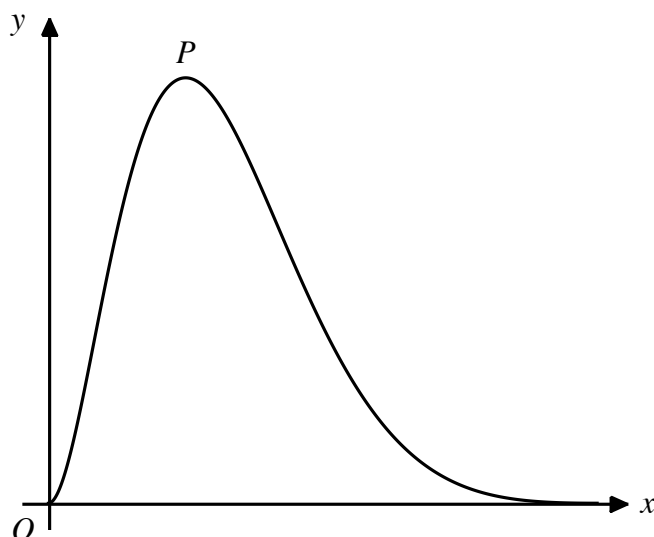
   (ii) Deduce that the radius of C_2 is $\sqrt{5}$.

(b) Find the coordinates of the centre of C_2.

(c) Write down the equation of the circle C_2, giving your answer in the form

$$(x + a)^2 + (y + b)^2 = k$$

where a, b and k are constants to be found.

4

The diagram above shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = 10\,e^{-2x}\sin^2 x, \quad 0 \leqslant x \leqslant \pi$$

(a) Explain why inverse of f does not exist.

The point P is a maximum point on C.

(b) Use calculus to show that the x coordinate of P satisfies

$$\cos x - \sin x = 0$$

(c) Find the exact coordinates of P.

(d) Deduce the range of the functions

    (i) $y = 3f(2x), \quad 0 \leqslant x \leqslant \dfrac{\pi}{2}$

    (ii) $y = f\left(x + \dfrac{\pi}{4}\right), \quad x \geqslant 0$

Pure questions – modelling

5 The population of a town, P, at the end of Year 1 is 30 000.

A model predicts that the population of the town will increase by 4% each year, forming a geometric sequence.

(a) Write down the common ratio of the geometric sequence.

(b) Find the population of the town at the end of Year 3.

The model predicts that the population will exceed 50 000 at the end of Year N.

(c) Showing your working clearly, find the value of N.

Year (n)	1	2	3	4	5
Population (P)	30 000	31 188	32 000	32 002	32 003

Another town has a population of 30 000 at the end of Year 1.

The size of this population at the end of the first five years is shown in the table above.

Craig wants to know if the same model can be used to model this town's population.

Using the data in the table,

(d) (i) evaluate use of the model for this town,

   (ii) suggest a possible explanation of the results.

Statistics and mechanics will be added later today