

Year 2 Pure Chapter 7: $a\sin x + b\cos x$ - Exam Questions (63 mins)

1.

The diagram above shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

- (a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha <$

(4)

- (b) Find the values of x , $0 \leq x < 2\pi$, for which $y = 1$.

(4)

(Total 8 marks)

2. (a) Express $5 \cos x - 3 \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0 < \alpha <$

(4)

- (b) Hence, or otherwise, solve the equation

$$5 \cos x - 3 \sin x = 4$$

for $0 \leq x < 2\pi$, giving your answers to 2 decimal places.

(5)

(Total 9 marks)

3.

$$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and ,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

4. $f(x) = 12 \cos x - 4 \sin x.$

Given that $f(x) = R \cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90^\circ$,

(a) find the value of R and the value of α .

(4)

(b) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for $0 \leq x \leq 360^\circ$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12 \cos x - 4 \sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

(Total 12 marks)

5. In a particular circuit the current, I amperes, is given by

$$I = 4 \sin \theta - 3 \cos \theta, \quad \theta > 0,$$

where θ is an angle related to the voltage.

Given that $I = R \sin (\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha < 360^\circ$,

- (a) find the value of R , and the value of α to 1 decimal place.

(4)

- (b) Hence solve the equation $4 \sin \theta - 3 \cos \theta = 3$ to find the values of θ between 0 and 360° .

(5)

- (c) Write down the greatest value for I .

(1)

- (d) Find the value of θ between 0 and 360° at which the greatest value of I occurs.

(2)

(Total 12 marks)