Write your name here

Surname	Other Names

AS/A Level Mathematics

Iteration

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.
• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
• **Fill in the boxes** at the top of this page with your name.
• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled..
• Answer the questions in the spaces provided
– there may be more space than you need.
• You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
• Answers should be given to three significant figures unless otherwise stated.

Information

• The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Try to answer every question.
• Check your answers if you have time at the end.

1 $f(x) = x^3 + 2x^2 + x + 1$

(a) Show that f(x) has a root between -1.5 and -2 **(2)**

(b) Show that the equation f(x) = 0 can be written in the form $x = -\left(\frac{x+1}{x^2} + 2\right)$ **(3)**

(c) Use the iteration formula $x_{n+1} = -\left(\frac{x_n + 1}{x_n^2} + 2\right)$ with x_0 = -2 to find, to 3 decimal places, the values of x_1, x_2 and x_3. **(3)**

The root of f(x) = 0 is α **(3)**

(d) By choosing a suitable interval, prove that α = -1.755 to 3 decimal places

(Total for question 1 is 11 marks)

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2 $f(x) = e^{x+2} + x - 10$

(a) Show that the equation f(x) = 0 can be written in the form $x = \ln(10 - x) - 2$ **(3)**

(b) Use the iteration formula $x_{n+1} = \ln(10 - x_n) - 2$ with x_0 = 0.5 to find, to 3 decimal places, the values of x_1, x_2 and x_3. **(3)**

The root of f(x) = 0 is α

(c) By choosing a suitable interval, prove that α = 0.275 to 3 decimal places **(3)**

(Total for question 2 is 9 marks)

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3 The curve C has the equation $y = (12 - x)\ln x$ $x > 0$

(a) Find $\frac{dy}{dx}$ **(3)**

(b) C has a stationary point at P. Show that the x coordinate of P lies between 4.5 and 5 **(2)**

(c) Show that the x coordinate of P is a solution of $x = \frac{12}{1 + \ln x}$ **(3)**

(d) Use the iteration formula $x_{n+1} = \frac{12}{1 + \ln x_n}$ with x_0 = 4.75 to find, **(3)**

to 3 decimal places, the values of x_1, x_2 and x_3.

(Total for question 3 is 11 marks)

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4 The curve C with the equation $y = e^x$ meets the line L with equation $y = 5x - 1$ at the point P.

(a) Show that the x coordinate of P satisfies $x = \ln(5x - 1)$ **(2)**

(b) Use the iteration formula $x_{n+1} = \ln(5x_n - 1)$ with x_0 = 2.5 to find, **(3)**

to 3 decimal places, the values of x_1, x_2 and x_3.

(Total for question 4 is 5 marks)