Worksheet 2 Solutions

Question 1 Solution.

(a) Using the binomial expansion, we have

$$
\begin{aligned}
(5x-2)^4 &= (5x)^4 + \binom{4}{1}(5x)^3(-2)^1 + \binom{4}{2}(5x)^2(-2)^2 + \binom{4}{3}(5x)^1(-2)^3 + (-2)^4 \\
&= 625x^4 - 1000x^3 + 600x^2 - 160x + 16
\end{aligned}
$$

Exam Tip: make sure the sum of the powers of each term equals the degree of the expansion.

(b) We use part (a) to obtain

$$
\begin{aligned}
\int_0^1 (5x-2)^4 dx &= \int_0^1 (625x^4 - 1000x^3 + 600x^2 - 160x + 16)dx \\
&= \left[\frac{625x^5}{5} - \frac{1000x^4}{4} + \frac{600x^3}{3} - \frac{160x^2}{2} + 16x\right]_0^1 \\
&= \left[125x^5 - 250x^4 + 200x^3 - 80x^2 + 16x\right]_0^1 \\
&= (125 - 250 + 200 - 80 + 16) - 0 \\
&= 11
\end{aligned}
$$

Question 2 Solution.

(a) This is a circle, so the coefficient of x^2 and y^2 needs to be the same. Hence $\boxed{a = 4}$.

(b) We complete the square:

$$\begin{aligned}
&4x^2 + 4y^2 - 5y = k - 2 \\
&\Rightarrow x^2 + y^2 - \frac{5}{4} = \frac{k-2}{4} \\
&\Rightarrow x^2 + \left(y - \frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2 = \frac{k-2}{4} \\
&\Rightarrow x^2 + \left(y - \frac{5}{8}\right)^2 = \frac{k-2}{4} + \frac{25}{64}
\end{aligned}$$

So the coordinates of the centre of C are $\boxed{(0, \frac{5}{8})}$.

(c) For this to be a circle, the radius must be positive, so

$$\begin{aligned}
&\frac{k-2}{4} + \frac{25}{64} > 0 \\
&\Rightarrow \frac{k-2}{4} > -\frac{25}{64} \\
&\Rightarrow k - 2 > -\frac{25}{16} \\
&\Rightarrow k > 2 - \frac{25}{16} = \frac{7}{16}
\end{aligned}$$

as required.

Question 3 Solution.

(a) The equation is equivalent to $16x^2 + 8kx + (8k - 15) = 0$. For the equation to have two real distinct roots, the discriminant must be positive, i.e.

$$(8k)^2 - 4(16)(8k - 15) > 0$$
$$\Rightarrow 64k^2 - 512k + 960 > 0$$

and then dividing by 64, we obtain $k^2 - 8k + 15 > 0$, as required.

(b) Consider $k^2 - 8k + 15 = 0 \Rightarrow (k - 5)(k - 3) = 0$, and so the equation has roots at $k = 3$ and $k = 5$. Drawing a graph will show you that the solutions are $\boxed{k < 3 \text{ and } k > 5}$.

Exam Tip: this question asks for the range of values of k, so you do not need to worry about set notation here. If the question asked for the 'set of values', then you would need to express your answer using set notation. In this case, the solution *set* could be expressed as,for example, $\{k \in \mathbb{R} : k < 3\} \cup \{k \in \mathbb{R} : k > 5\}$.

(c) Note that this equation comes from letting $k = 2$, so it must have $\boxed{\text{two distinct real roots}}$ by our working in (b).

Question 4 Solution.

(a) At time $t = 0$, we have $v = -k$. The magnitude of v is 4, which means $k = \pm 4$, but since k is positive, $\boxed{k = 4}$.

(b) Acceleration is the derivative of velocity with respect to time, so

$$\left.\frac{dv}{dt}\right|_{t=2} = 2t|_{t=2} = 2(2) = 4$$

$\therefore$ The acceleration of P at $t = 2$ is $\boxed{4 \text{ m/s}^2}$.

(c) The area under the velocity-time graph between $0 \leq t \leq 5$ is the distance travelled by the particle in its first five seconds of motion. We can find this area using integration. However, we have to be careful since the graph of $v = t^2 - 4$ is below the x-axis for $0 \leq t \leq 2$:

$$\begin{aligned}
\text{Distance travelled} &= -\int_0^2 v dt + \int_2^5 v dt \\
&= -\int_0^2 (t^2 - 4)dt + \int_2^5 (t^2 - 4)dt \\
&= -\left[\frac{t^3}{3} - 4t\right]_0^2 + \left[\frac{t^3}{3} - 4t\right]_2^5 \\
&= -\left(\frac{2^3}{3} - 4(2) - 0\right) + \left(\frac{5^3}{3} - 4(5) - \frac{2^3}{3} + 4(2)\right) \\
&= \frac{16}{3} + \frac{65}{3} + \frac{16}{3} \\
&= \frac{97}{3} \text{ m}
\end{aligned}$$

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