AS-Level Mathematics - Pure Maths Test - 'BASIC'

1. (a) **Factorise** $6x^2 + 7x - 3$

   Two numbers that multiply to make 6 (in the $6x^2$) could be 2, 3 or 6, 1. Two numbers that multiply to make the -3 at the end could be -1, 3 or 1, -3. Try combinations until the correct expansion is obtained:

$$\begin{aligned}(6x-1)(x+3) &= 6x^2+17x-3 \quad \times\\ (2x-1)(3x+3) &= 6x^2+3x-3 \quad \times\\ (6x+1)(x-3) &= 6x^2-17x-3 \quad \times\\ (2x+3)(3x-1) &= 6x^2+7x-3\end{aligned}$$

   (b) **Simplify** $(9x^2)^{\frac{3}{2}}$

$$\begin{aligned}\left(9x^2\right)^{\frac{3}{2}} &= 9^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}\\ &= \left(9^{\frac{1}{2}}\right)^3\left(\left(x^2\right)^{\frac{1}{2}}\right)^3\\ &= (3)^3(x)^3\\ &= 27x^3\end{aligned}$$

   (c) **Express** $2\sqrt{20}+\sqrt{45}$ **in the form** $a\sqrt{5}$ **where** a **is an integer.**

$$\begin{aligned}2\sqrt{20}+\sqrt{45} &= 2\sqrt{4\times 5}+\sqrt{9\times 5}\\ &= 2\sqrt{4}\sqrt{5}+\sqrt{9}\sqrt{5}\\ &= 2\times 2\times\sqrt{5}+3\times\sqrt{5}\\ &= 4\sqrt{5}+3\sqrt{5}\\ &= 7\sqrt{5}\end{aligned}$$

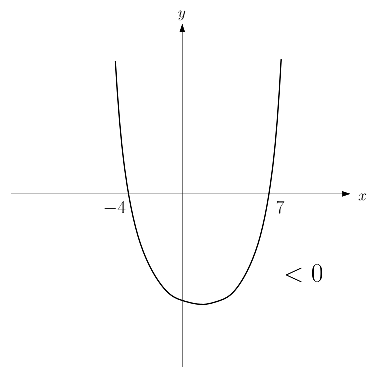
2. **Find the values of** k **for which the quadratic equation** $(k-1)x^2+3kx+k-1$ **has a single repeated root**.

   A single repeated root implies that the discriminant, b^2-4ac, should be 0 where $a=k-1$, $b=3k$ and $c=k-1$. Hence,

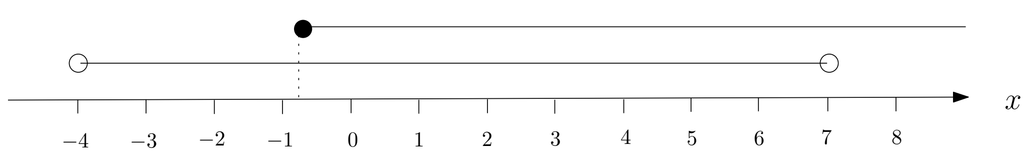
$$\begin{aligned}&(3k)^2-4(k-1)(k-1)=0\\ \Rightarrow\quad &9k^2-4\left(k^2-2k+1\right)=0\\ \Rightarrow\quad &9k^2-4k^2+8k-4=0\\ \Rightarrow\quad &5k^2+8k-4=0\\ \Rightarrow\quad &(5k-2)(k+2)=0\\ \Rightarrow\quad &k=\frac{2}{5},\quad k=-2\end{aligned}$$

3. **Find the set of x values for which $x^2 - 3x < 28$ AND $4x + 3 \geq 0$.**

The first inequality gives $x^2 - 3x - 28 < 0$ or $(x-7)(x+4) < 0$. Plot the graph of $y = (x-7)(x+4)$ and look for where it is less than 0:

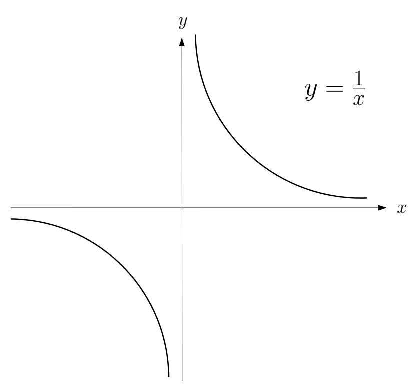


The graph is less than zero when x is between -4 and 7, i.e. $-4 < x < 7$. It does not include -4 and 7 since this is where the function is exactly equal to zero.
The second inequality gives $4x \geq -3$ or $x \geq \frac{-3}{4}$. Both inequalities must hold simultaneously:

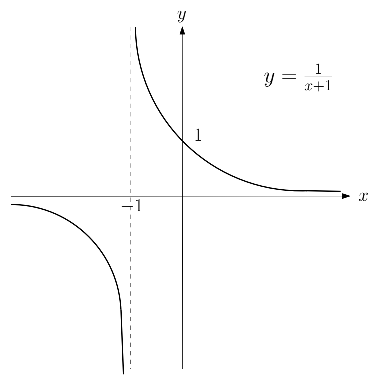


Hence, both inequalities hold when $\frac{-3}{4} \leq x < 7$.

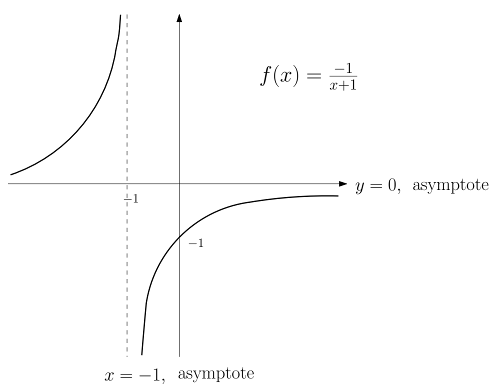
4. **Sketch the graph of $f(x) = \frac{-1}{x+1}$, labelling any intersections and asymptotes.**
The graph of $\frac{-1}{x+1}$ can be found by performing two transformations of the curve $\frac{1}{x}$. The graph of $\frac{1}{x}$ is given by:



The graph of $\frac{1}{x+1}$ is found by replacing x by $x+1$, i.e by moving the graph of $\frac{1}{x}$ to the left by 1:



Make this graph negative, i.e. reflect in the x-axis, then we obtain the graph of $f(x) = \frac{-1}{x+1}$:



5. **Simplify the following fractions:**

   (a) $\frac{x^2-7x+12}{x-4}$

   (b) $\frac{2x^3+13x^2+16x-15}{x+3}$

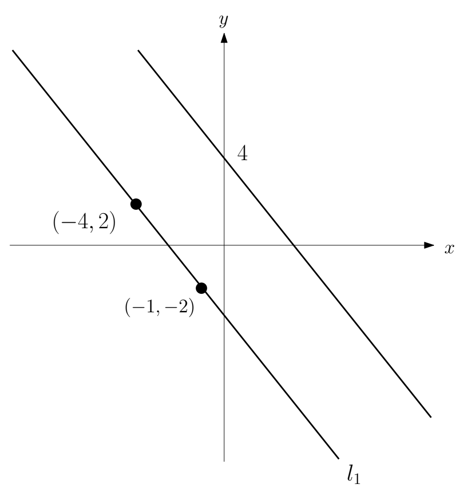
(a) $\frac{x^2-7x+12}{x-4} = \frac{(x-4)(x-3)}{(x-4)} = x - 3$

(b)

$$\begin{array}{r|l} & 2x^2 \quad +7x \quad -5 \\ \hline x+3 & 2x^3+13x^2+16x-15 \\ & -2x^3 \quad -6x^2 \\ \hline & \qquad 7x^2+16x \\ & \qquad -7x^2-21x \\ \hline & \qquad\qquad -5x-15 \\ & \qquad\qquad 5x+15 \\ \hline & \qquad\qquad\qquad 0 \end{array}$$

Note that some of the rows have been made negative so that they are being added together instead of being subtracted. You may use the method you prefer as long as you get the right answer.

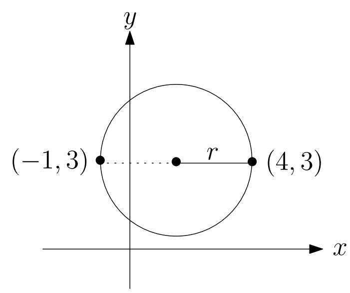
6. **Line l_1 passes through the points $(-4, 2)$ and $(-1, -2)$. Find the equation of the line parallel to l_1 that intersects the y-axis at $y = 4$.**



The gradient of the line l_1 is given by $m = \frac{\Delta y}{\Delta x} = \frac{-2-2}{-1-(-4)} = \frac{-4}{3}$. The line required is parallel to l_1 and so also has gradient $\frac{-4}{3}$. This line has y-intercept 4 and so the equation is $y = \frac{-4}{3}x + 4$.

7. **Find the equation of the circle whose diameter is the straight line between $(-1, 3)$ and $(4, 3)$.**

The equation of a circle is given by $(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the centre of the circle and r is the radius.



Sketching the information, one can identify the centre of the circle at the point $\left(\frac{3}{2}, 3\right)$. The radius is clearly given by $r = \frac{5}{2}$ and so the equation of the circle is given by

$$\left(x - \frac{3}{2}\right)^2 + (y-3)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}.$$

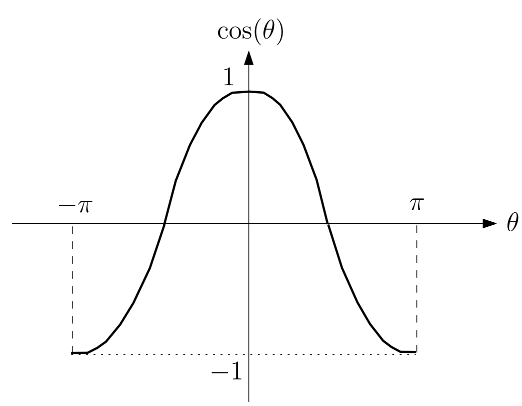
8. **Find the first 3 terms, in ascending powers of x, of the expansion $(2x - y)^5$.**
The first three terms in the expansion of $(a+b)^n$ as given by the Edexcel Formula Booklet are:

$$(a+b)^n \approx a^n + \begin{pmatrix} n \\ 1 \end{pmatrix} a^{n-1}b + \begin{pmatrix} n \\ 2 \end{pmatrix} a^{n-2}b^2$$

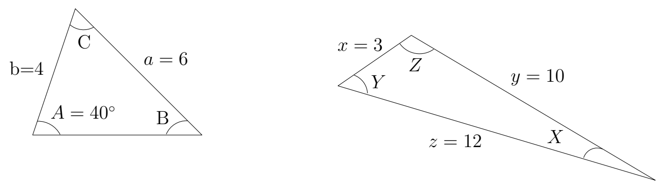
Swapping n for 5, a for $-y$ and b for $2x$ (so that powers of x are ascending) we have

$$\begin{aligned} (-y+2x)^5 &= (-y)^5 + \begin{pmatrix} 5 \\ 1 \end{pmatrix} (-y)^4(2x) + \begin{pmatrix} 5 \\ 2 \end{pmatrix} (-y)^3(2x)^2 \\ &= -y^5 + 5 \times y^4 \times 2x + 10 \times -y^3 \times 4x^2 \\ &= -y^5 + 10y^4x - 40y^3x^2 \end{aligned}$$

9. **Sketch the graph of $\cos(x)$ between $-\pi$ and π.**



10. **Find the missing angles of these triangles.**



Labelling the first triangle appropriately and using the sine rule: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ we have

$$\frac{\sin(40)}{6} = \frac{\sin(B)}{4} \quad \Rightarrow \quad B = \sin^{-1}\left(4 \times \frac{\sin(40)}{6}\right) = 35.26° \text{ to 2 d.p.}$$

The remaining angle, C, in the first triangle is given by

$$180° - 40° - 35.26° = 104.74° \text{ to 2 d.p.}$$

Using the cosine rule on the second triangle, we have

$$\begin{aligned} x^2 &= y^2 + z^2 - 2yz\cos(X) \\ \Rightarrow \quad 3^2 &= 10^2 + 12^2 - 2 \times 10 \times 12 \times \cos(X) \\ \Rightarrow \quad 9 &= 100 + 144 - 240\cos(X) \\ \Rightarrow \quad -235 &= -240\cos(X) \\ \Rightarrow \quad X &= \cos^{-1}\left(\frac{-235}{-240}\right) = 11.72° \end{aligned}$$

Following with the sine rule we obtain angle Y:

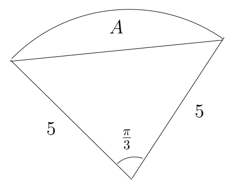
$$\frac{\sin(11.72)}{3} = \frac{\sin(Y)}{10} \quad \Rightarrow \quad Y = \sin^{-1}\left(10 \times \frac{\sin(11.72)}{3}\right) = 42.60° \text{ to 2 d.p.}$$

The remaining angle, Z, in the second triangle is given by

$$180° - 11.72° - 42.60° = 125.68° \text{ to 2 d.p.}$$

Be careful to maintain accuracy throughout your calculations, rounding errors can get quite large when dealing with trigonometric functions.

11. **Find the area of the region marked A:**

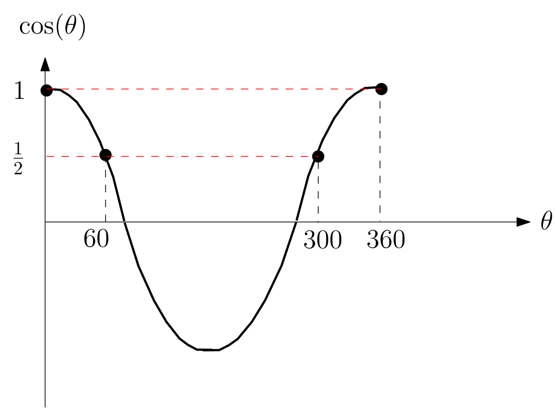


In order to answer this question you must know the formula for the area of a triangle, $\frac{1}{2}ab\sin(C)$, and the formula for the area of a sector, $\frac{1}{2}r^2\theta$, where θ is measured in radians. Subtracting the area of the triangle from the area of the sector we have the area of region A given by:

$$\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5 \times 5 \times \sin\left(\frac{\pi}{3}\right) = 2.265 \text{ to 3 d.p.}$$

12. **Solve the equation** $2\sin^2(\theta) + 3\cos(\theta) = 3$ **on the interval** $0° \leq \theta \leq 360°$**.**

$$\begin{aligned}
& 2\sin^2(\theta) + 3\cos(\theta) = 3 \\
\Rightarrow \quad & 2\left(1 - \cos^2(\theta)\right) + 3\cos(\theta) = 3 \\
\Rightarrow \quad & 2 - 2\cos^2(\theta) + 3\cos(\theta) = 3 \\
\Rightarrow \quad & -2\cos^2(\theta) + 3\cos(\theta) - 1 = 0 \\
\Rightarrow \quad & 2\cos^2(\theta) - 3\cos(\theta) + 1 = 0 \\
\Rightarrow \quad & (2\cos(\theta) - 1)(\cos(\theta) - 1) = 0 \\
\Rightarrow \quad & 2\cos(\theta) - 1 = 0, \quad \cos(\theta) - 1 = 0 \\
\Rightarrow \quad & \cos(\theta) = \frac{1}{2} \quad \text{OR} \quad \cos(\theta) = 1
\end{aligned}$$



The values of θ between $0°$ and $360°$ that give either $\cos(\theta) = 1$ or $\cos(\theta) = \frac{1}{2}$ are

$$\theta = 0°, 60°, 300°, 360°.$$

13. Find the values of the following logarithms without a calculator:

(a) $\log_4(64)$

(b) $\log_3\left(\frac{1}{9}\right)$

(c) $\log_8(1)$

(a) $\log_4(64)$ can be read as '*the power of 4 that gives 64*' and we know this is 3 since $4^3 = 64$. Hence, $\log_4(64) = 3$.

(b) $\log_3\left(\frac{1}{9}\right)$ can be read as '*the power of 3 that gives* $\frac{1}{9}$' and this is -2 since $3^{-2} = \frac{1}{9}$. Hence, $\log_3\left(\frac{1}{9}\right) = -2$.

(c) $\log_8(1)$ can be read as '*the power of 8 that gives 1*' and this is 0 since $8^0 = 1$, in fact, anything to the power of 0 is 1. Hence, $\log_8(1) = 0$.

14. **Given that** $y = \frac{1-2x^3}{x^2}$

(a) Find $\frac{dy}{dx}$

(b) Find $\frac{d^2y}{dx^2}$

y can be written as $\frac{1}{x^2} - \frac{2x^3}{x^2} = x^{-2} - 2x$. In this format, one can easily find the required derivatives:

(a) $\frac{dy}{dx} = -2x^{-3} - 2 = \frac{-2}{x^3} - 2$

(b) $\frac{d^2y}{dx^2} = 6x^{-4} = \frac{6}{x^4}$

15. **On what interval is the function** $x^2 + 4x - 1$ **a decreasing function?**

A function is decreasing when it is downward sloping, i.e. when it has a negative gradient. The derivative is given by $2x + 4$. This is negative when

$$2x + 4 < 0 \quad \Rightarrow \quad x < -2$$

Hence, $x^2 + 4x - 2$ is decreasing on the interval $x < -2$.

16. **Find the following integrals:**

(a) $\int x^{-3}\, dx$

(b) $\int \left(5x^{\frac{3}{4}} - 2\right) dx$

(c) $\int \frac{1}{\sqrt{x}} \, dx$

(a) $\int x^{-3} \, dx = \frac{x^{-2}}{-2} + c = -\frac{1}{2x^2} + c$

(b) $\int \left(5x^{\frac{3}{4}} - 2\right) dx = \frac{5x^{\frac{7}{4}}}{\frac{7}{4}} - 2x + c = \frac{20x^{\frac{7}{4}}}{7} - 2x + c$

(c) $\int \frac{1}{\sqrt{x}} \, dx = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = 2x^{\frac{1}{2}} + c = 2\sqrt{x} + c$

17. **Evaluate** $\int_1^2 3x \, dx$.

$$\begin{aligned} \int_1^2 3x \, dx &= \left[\frac{3x^2}{2}\right]_1^2 \\ &= \frac{3(2)^2}{2} - \frac{3(1)^2}{2} \\ &= \frac{12}{2} - \frac{3}{2} \\ &= \frac{9}{2} \end{aligned}$$

