Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics
Further Mechanics 1
Practice Paper 3

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A particle P of mass 0.75 kg is moving with velocity $4\mathbf{i}$ m s^{-1} when it receives an impulse $(6\mathbf{i} + 6\mathbf{j})$ N s. The angle between the velocity of P before the impulse and the velocity of P after the impulse is $\theta°$.

Find

(a) the value of θ,

(5)

(b) the kinetic energy gained by P as a result of the impulse.

(3)

(Total 8 marks)

[Mark scheme for Question 1]

[Examiner comment]

2.

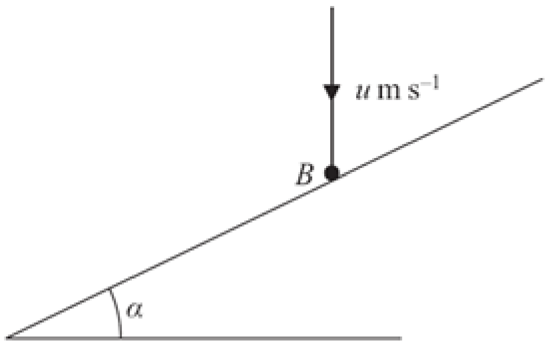


Figure 1

A smooth fixed plane is inclined at an angle α to the horizontal. A smooth ball B falls vertically and hits the plane. Immediately before the impact the speed of B is u m s^{-1}, as shown in Figure 1. Immediately after the impact the direction of motion of B is horizontal. The coefficient of restitution between B and the plane is $\frac{1}{3}$.

Find the size of angle α.

(Total 6 marks)

[Mark scheme for Question 2]

[Examiner comment]

---

3. A particle P of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When P has moved 12 m, its speed is 4 m s^{-1}. Given that friction is the only non-gravitational resistive force acting on P, find

(a) the work done against friction as the speed of P increases from 0 m s^{-1} to 4 m s^{-1},

(4)

(b) the coefficient of friction between the particle and the plane.

(4)

(Total 8 marks)

[Mark scheme for Question 3]

[Examiner comment]

---

4. A cyclist and her bicycle have a total mass of 70 kg. She cycles along a straight horizontal road with constant speed 3.5 m s^{-1}. She is working at a constant rate of 490 W.

(a) Find the magnitude of the resistance to motion.

(4)

The cyclist now cycles down a straight road which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$, at a constant speed U m s^{-1}. The magnitude of the non-gravitational resistance to motion is modelled as $40U$ newtons. She is now working at a constant rate of 24 W.

(b) Find the value of U.

(7)

(Total 11 marks)

[Mark scheme for Question 4]

[Examiner comment]

---

5. Two particles A and B, of mass $2m$ and $3m$ respectively, are initially at rest on a smooth horizontal surface. Particle A is projected with speed $3u$ towards B. Particle A collides directly with particle B. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After the collision B hits a fixed smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is e. The magnitude of the impulse received by B when it hits the wall is $\frac{27}{4}mu$.

(b) Find the value of e.

(3)

(c) Determine whether there is a further collision between A and B after B rebounds from the wall.

(2)

(Total 12 marks)

[Mark scheme for Question 5]

[Examiner comment]

6. A smooth uniform sphere S is moving on a smooth horizontal plane when it collides obliquely with an identical sphere T which is at rest on the plane. Immediately before the collision S is moving with speed U in a direction which makes an angle of 60° with the line joining the centres of the spheres. The coefficient of restitution between the spheres is e.

(a) Find, in terms of e and U where necessary,

(i) the speed and direction of motion of S immediately after the collision,

(ii) the speed and direction of motion of T immediately after the collision.

(12)

The angle through which the direction of motion of S is deflected is δ°.

(b) Find

(i) the value of e for which δ takes the largest possible value,

(ii) the value of δ in this case.

(3)

(Total 15 marks)

Mark scheme for Question 6

Examiner comment

7. A particle P of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons. The ends of the string are attached to two fixed points A and B, where AB is horizontal and $AB = 0.48$ m. Initially P is held at rest at the mid-point, M, of the line AB and the tension in the string is 240 N.

(a) Show that $\lambda = 400$.

(3)

The particle is now held at rest at the point C, where C is 0.07 m vertically below M. The particle is released from rest at C.

(b) Find the magnitude of the initial acceleration of P.

(6)

(c) Find the speed of P as it passes through M.

(6)

(Total 15 marks)

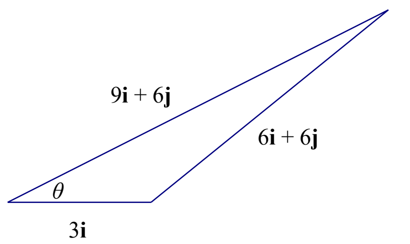
Mark scheme for Question 7

Examiner comment

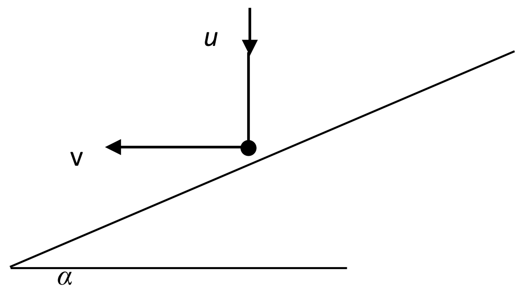
TOTAL FOR PAPER: 75 MARKS

Further Mathematics – Further Mechanics 1– Practice Paper 03 – Mark scheme –

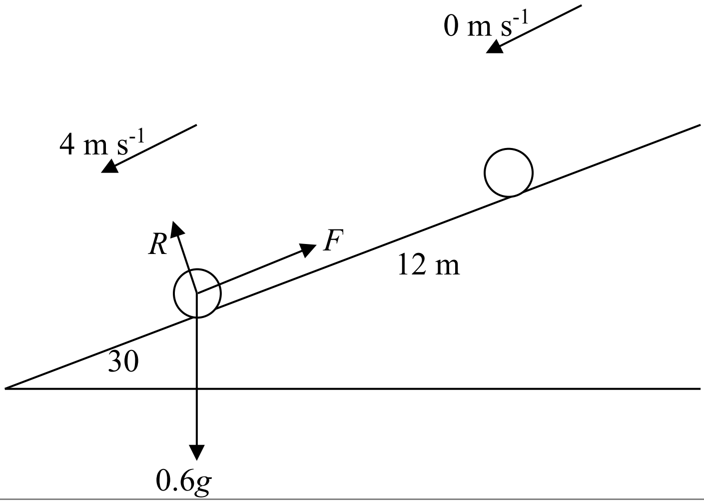
Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	$0.75\mathbf{v} = 6\mathbf{i} + 6\mathbf{j} + 0.75 \times 4\mathbf{i} \; (= 9\mathbf{i} + 6\mathbf{j})$	M1A1
	$\mathbf{v} = 12\mathbf{i} + 8\mathbf{j}$	A1
		
	$\theta = \tan^{-1}\left(\frac{2}{3}\right)$ or $\theta = \cos^{-1}\left(\frac{1+13-8}{2\sqrt{13}}\right)$, or equivalent	M1
	$33.7°$ or 0.588 radians	A1
		(5)
(b)	Change in KE $= \frac{1}{2} \times \frac{3}{4}(144+64) - \frac{1}{2} \times \frac{3}{4}(16)$	M1
	$= 72$ (J)	A1ft A1
		(3)
		(8 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2		
	CLM: $u\sin\alpha = v\cos\alpha$	M1A1
	Impact: $\frac{1}{3}u\cos\alpha = v\sin\alpha$	M1A1
	$\frac{1}{3}\times\frac{1}{\tan\alpha} = \tan\alpha$	M1
	$\tan\alpha = \frac{1}{\sqrt{3}}$	
	$\alpha = 30°\ \left(\text{or } \frac{\pi}{6} \text{ or } 0.52\,\text{rad}\right)$	A1
		(6)
		(6 marks)

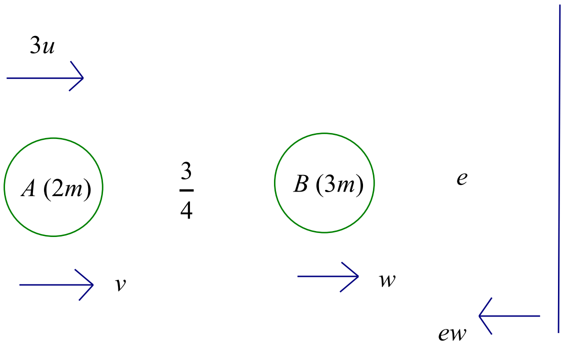
Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)		
	K.E gained $= \frac{1}{2}\times 0.6\times 4^2$	
	P.E. lost = $0.6\times g\times(12\sin 30)$	
	Change in energy = P.E. lost - K.E. gained	
	$= 0.6 \times g \times 12\sin 30 - \frac{1}{2} \times 0.6 \times 4^2$	M1A1
	$= 30.48$	A1
	Work done against friction = 30 or 30.5 J	A1
		(4)
(b)	R$(\uparrow)$ $R = 0.6g\cos 30$	B1
	$F = \dfrac{30.48}{12}$	B1ft
	$F = \mu R$	
	$\mu = \dfrac{30.48}{12\times 0.6g\cos 30}$	M1
	$\mu = 0.4987$	
	$\mu = 0.499$ or 0.50	A1
		(4)
		(8 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4(a)	$\frac{490}{3.5} - R = 0$	**B1M1 A1**
	$R = 140$ N	**A1**
		(4)
(b)	$\frac{24}{u} + 70g.\frac{1}{14} - 40u = 0$	**B1**
	$40u^2 - 49u - 24 = 0$	**M1A2 ,1,0**
	$(5u - 8)(8u + 3) = 0$	**DM1**
	$u = 1.6$	**DM1 A1**
		(7)
		(11 marks)

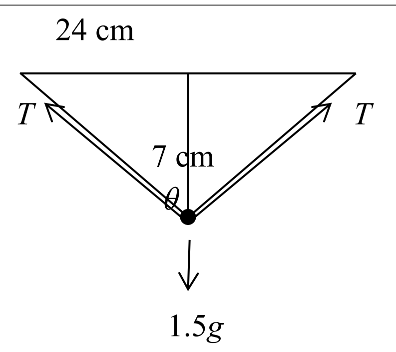
Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)		
	CLM: $6mu = 2mv + 3mw$	M1
	$(6u = 2v + 3w)$	A1
	Impact: $w - v = \frac{3}{4} \times 3u \left(= \frac{9}{4}u \right)$	M1A1
	$6u = 2w - \frac{9}{2}u + 3w$	DM1
	$w = \frac{21}{10}u = v_B$	A1
	$v = w - \frac{9}{4}u = \left(\frac{21}{10} - \frac{9}{4} \right)u = -\frac{3}{20}u$, $v_A = \frac{3}{20}u$	A1
		(7)
(b)	Speed of B after hitting wall $= \frac{21}{10}ue$	M1
	Impulse $= \frac{27}{4}mu = 3m\left(\frac{21}{10}u + \frac{21}{10}ue \right)$	M1
	$\frac{9}{4} = \frac{21}{10}(1+e)$, $e = \frac{1}{14}$	A1
		(3)
(c)	Speed of B after second impact $= \frac{1}{14} \times \frac{21}{10}u = \frac{3}{20}u$	B1ft
	Same velocity (and A has a head start), so no collision.	B1
		(2)
		(12 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme		Marks
6(a)	$mv_1 + mv_2 = mu\cos 60^\circ$	Momentum	M1A1
	$-v_1 + v_2 = eu\cos 60^\circ$	Impact law	M1A1
	$v_1 = \dfrac{u(1-e)}{4}$		M1A1
	Speed of $S = \sqrt{\dfrac{u^2(1-e)^2}{16} + \dfrac{3u^2}{4}}$	Solve for v_1 and find speed	A1
	$= \dfrac{u}{4}\sqrt{e^2 - 2e + 13}$		M1A1
	$\tan\theta = \dfrac{u\sqrt{3}}{2v_1} = \dfrac{2\sqrt{3}}{(1-e)}$	Use components to find dirn	
	S moves at arctan $\dfrac{2\sqrt{3}}{(1-e)}$ to the line of centres		M1A1
	$v_2 = \dfrac{u(1+e)}{4}$	v_2 in terms of u, e	B1
			(12)
(b)	T has speed $\dfrac{u(1+e)}{4}$ along the line of centres	Conclusion	
	θ is a max when $e = 1$ then $\theta = 90^\circ$ then deflection angle is $90^\circ - 60^\circ = 30^\circ$		M1A1
	$\delta = 30$		A1
			(3)
			(15 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(a)	$T=\frac{\lambda x}{l} \Rightarrow 240=\frac{\lambda \times 18}{30}$	M1A1
	$\lambda = 400$	A1
		(3)
(b)		
	Extension = 10 cm or 20 cm (used in (b) or (c))	B1
	$T=\frac{400\times 10}{15}=\left(\frac{800}{3}\right)$	M1 A1ft
	R$(\uparrow)$ $2T\cos\theta - 1.5g = (\pm)1.5a$	M1A1
	$\frac{1600}{3}\times\frac{7}{25}-1.5\times 9.8=(\pm)1.5a$	
	$a=89.75...$ $a=90$ m s^{-2} or 89.8 (positive)	A1
		(6)
(c)	E.P.E. $=\frac{1}{2}\times 400\times\frac{0.2^2}{0.3}$	B1ft
	$1.5g\times 0.07+\frac{1}{2}\times 1.5v^2 = 200\times\frac{0.2^2}{0.3}-\frac{200\times 0.18^2}{0.3}$	M1A1 A1
	$v^2=\frac{1}{0.75}\left(200\times\frac{0.2^2}{0.3}-\frac{200\times 0.18^2}{0.3}-1.5g\times 0.07\right)$	M1 dep
	$v=2.32...=2.3$ m s^{-1}	A1
		(6)
		(15 marks)

Further Mathematics – Further Mechanics 1 – Practice Paper 03 – Examiner Report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. (a) The majority of candidates used the impulse-momentum equation correctly, although some of the solutions were quite laboured. The incorrect form $\mathbf{I} = m\mathbf{u} - m\mathbf{v}$ was a common error, but most errors at this stage were in dividing by 0.75.

Some candidates used scalar product to find the angle between the two velocities, but the majority used $\tan^{-1}\left(\frac{8}{12}\right)$. Several candidates went on to subtract the required angle from 180°, suggesting some confusion about the angle between the two vectors.

(b) Many candidates answered this correctly. Most candidates were able to find a difference between the kinetic energy terms with the majority of candidates correctly finding the magnitude of their velocity vector. The most common error was to find the magnitude of the change in momentum (using v rather than v^2), but the incorrect form $\frac{1}{2}m(v-u)^2$ was also quite common.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. Many fully correct solutions were seen. When things did go wrong it was usually because the candidate applied conservation of linear momentum perpendicular to the plane and the impact law parallel to the plane.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. This question proved to be straightforward for well-prepared candidates.

In part (a) it was pleasing to see many candidates tackling this using the work-energy method, and there was less evidence this year of candidates double counting by including both the change in GPE and the work done against the weight, but candidates sometimes confused work done with just potential energy lost, or just kinetic energy gained. The alternative method using *suvat* to find the acceleration and then using $F = ma$ was also common. In the final answers there was considerable confusion between work done against friction and the frictional force. Many lost the final A mark by leaving the answer as 30.48 despite having used $g = 9.8$.

In part (b) candidates frequently did not make the connection with part (a) and proceeded to start again from scratch. In this case, a common but expensive error was to omit the component of the weight from their equation of motion.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. Very few students were unable to find the magnitude of the resistance to motion in (a) although some did produce some lengthy arguments before achieving the required solution. Others omitted to justify that the resistance had the same magnitude as the driving force.

In part (b) most candidates were able to attempt the equation of motion, although some failed to notice or to take correct account of the fact that this cyclist is moving down the road, rather than up, resulting in several sign errors. Most candidates were able to manipulate the equation, successfully incorporating $F = \frac{24}{u}$ (or equivalent) and going on to obtain and solve a quadratic equation.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. Part (a) was a standard question of its type and setting up the momentum and impact equations was very well done with velocities consistent between the two equations and very few sign errors. Most candidates found the correct solution to the simultaneous equations with just a few making manipulation errors. The most common error was to give a negative value for the speed of A, having found a negative value for the velocity.

In part (b) many candidates recognised that they needed to consider the effect of the impact with the wall and to form an impulse-momentum equation with $3m$. The most common error was confusion in the impulse-momentum equation between the directions of the impulse and the approach velocity of B. As a result of this or other errors many candidates found a value of e greater than 1 but did not appear to spot that this must be incorrect. Those candidates whose work led to the velocity after impact with the wall being greater than the velocity before, or the particle continuing to move towards the wall after impact should have recognised that they must have made an error and checked back through their work. The working in this part of the question was often difficult to follow because u disappeared and reappeared from one line to the next - this is something that a candidate should be able to spot when they check through their work.

In part (c) a good comparison of the speeds of A and B was usually made, although it is sensible to quote the two speeds at this stage and not expect the reader to turn back to find the result of part (a). The best candidates were more explicit about the direction in which the particles were travelling, and compared velocities, not just speeds.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. Those students who started with a clear diagram showing the velocities of both spheres before and after the collision worked through part (a) with few problems other than algebraic slips. They all understood that if T was at rest before the collision then it would move along the line of centres.

Part (b) required the students to recognise that the largest value of δ occurs when θ has the largest possible value. The majority of students made little progress with this part.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. In Q7(a) of the question all but the weakest candidates were able to score full marks for a simple application of Hooke's Law.

Q7(b) was a slightly more complex problem which required the candidates to set up the problem carefully. Most were able to identify the extension of either the string or half-string and the trigonometry did not pose a difficulty for most of the candidates. There were some errors in applying Newton's Second Law with incorrect resolutions and, occasionally, a single term in T. Those who were successful in setting up the problem usually managed to arrive at the correct answer although a number retained too many significant figures in their final result.

In Q7(c) a substantial proportion of candidates assumed that the Elastic Potential Energy was automatically zero in one of the two positions examined; most candidates were able find correct expressions for the other energy components. The majority of those who set up the energy equation successfully found little difficulty in arriving at the correct final result.