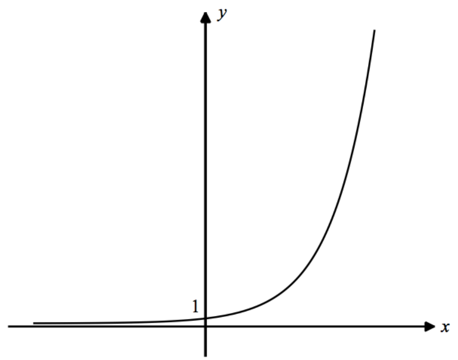
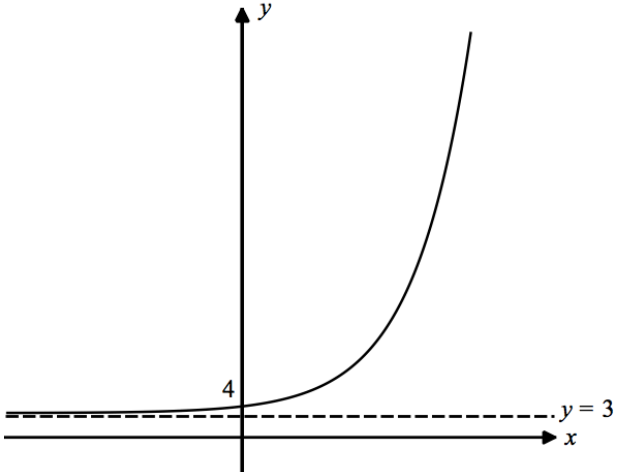
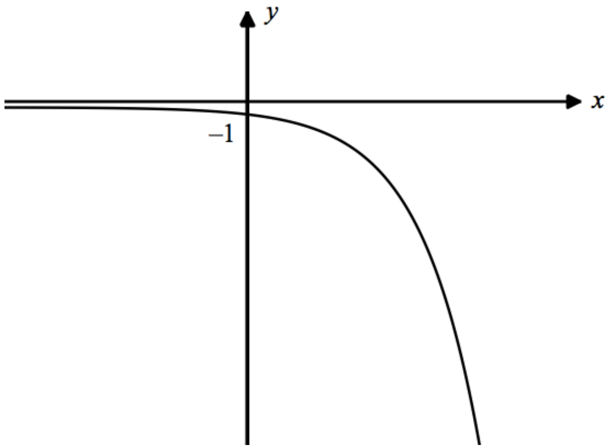
AS Level
Maths

Bronze Set B, Paper 1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1	$3^7+\binom{7}{1}(3)^6\left(\frac{x}{2}\right)^1+\binom{7}{2}(3)^5\left(\frac{x}{2}\right)^2+\binom{7}{3}(3)^4\left(\frac{x}{2}\right)^3+\ldots$ $=2187+\frac{5103}{2}x+\frac{5103}{4}x^2+\frac{2835}{8}x^3+\ldots$	B1 M1 A1 A1 A1 oe **[5]**	For 3^7 term appearing anywhere in expansion For at least one term of the form ${}^7C_k\,(3)^k\,(x/2)^{7-k}$, $k \neq 0, 7$ oe At least two of the required terms correct, unsimplified or better All four of the required terms correct, unsimplified or better Correct expansion with all terms simplified oe **MR:** If candidate gives first four terms in descending powers of x, treat as a misread
2	$2\cos\theta=5\sin\theta$ $\Rightarrow \frac{\sin\theta}{\cos\theta}=\frac{2}{5}$ $\Rightarrow \tan\theta=\frac{2}{5}$ Principal angle is 21.801… Others are $21.801...+180$, $21.801-180$ and $21.801-360$ So solutions are 21.8, 201.8, –158.2 and –338.2	B1 B1ft M1 A1 cao **[4]**	Re-arranges equation to the form $\tan\theta=k$ Correct principal angle ft their k Correct method to findat least one other solution in range ft their equation Correct solutions cao
3 (a)		B1 B1	Correct shape Correct intersection point at (0, 1) marked and equation of the asymptote clearly stated, shown or indicated on diagram (or equivalent, e.g. 'x axis')

	(one) asymptote at $y = 0$	**[2]**	
3 (b)	 (one) asymptote at $y = 3$	B1ft B1ft **[2]**	Correct shape ft their (a) Correct intersection point at (0, 4) marked and equation of the asymptote stated or shown on diagram ft their (a) Asymptote does not need to be shown or labelled on sketch If the asymptote is drawn on the sketch **and** labelled, and the candidate does not explicitly identify it as the asymptote, award B1 BOD
3 (c)	 (one) asymptote at $y = 0$	B1ft B1ft **[2]**	Correct shape Correct intersection point at (0, –1) marked and equation of the asymptote clearly stated, shown or indicated on diagram (or equivalent, e.g. ‘x axis’)

4 (a)	$\sqrt{(4-2)^2+(2--1)^2}=\sqrt{13}$	M1 A1 **[2]**	Uses formula for distance between two points/Pythagoras' Theorem Correct radius
4 (b)	$(x-2)^2+(y+1)^2=13$	B1 B1ft **[2]**	LHS correct RHS correct ft their radius in (a)
4 (c)	$(-2)^2+(y+1)^2=13$ $\Rightarrow y^2+2y-8=0$ $\Rightarrow (y-2)(y+4)=0$ so B has coordinates (0, –4)	M1 A1 A1 **[3]**	Substitutes $x = 0$ into the equation of the circle Obtains correct 3TQ Correct coordinates of B identified
4 (d)	$\frac{-4--1}{0-2}=\frac{3}{2}$, so gradient of tangent at B is $-\frac{2}{3}$ so equation of tangent to C at B is $y=-\frac{2}{3}x-4$	M1 A1 A1 **[3]**	Complete method to find the gradient of the tangent to C Correct gradient of the tangent Correct equation of the tangent in any form
5 (a)	$\int_{-2}^{x}(2p+4)dp=\left[\frac{2p^2}{2}+4p\right]_{-2}^{x}$ $=x^2+4x-(4-8)$ $=x^2+4x+4$	M1* A1 M1(dep*) A1 **[4]**	Attempts to integrate at least one term indefinitely Correct indefinite integration of both terms Substitutes limits into their integral in the correct order Obtains the correct quadratic convincingly with no errors seen
5 (b)	$x^2+4x+4=(x+2)^2\geq 0$ since any square number is always non-negative	B1* B1(dep*) **[2]**	Obtains $(x + 2)^2$ Explains why $(x + 2)^2$ is greater than or equal to 0. Allow "it is a square number", "any square number is negative", "the curve $y = (x + 2)^2$ is U shaped with minimum at (–2, 0)"
6 (a)	$\cos\theta=\frac{6^2+7^2-8^2}{2(6)(7)}\Rightarrow\cos\theta=\frac{1}{4}$	M1 A1 **[2]**	Uses the cosine rule (formula must be correct but in any form) Obtains correct result

6 (b)	$\sin\theta = \dfrac{\sqrt{15}}{4}$ $2R\left(\dfrac{\sqrt{15}}{4}\right) = 8$ $\Rightarrow R = \dfrac{16}{\sqrt{15}} = \dfrac{16\sqrt{15}}{15}$	B1 M1 A1 **[3]**	Correct value of $\sin\theta$ (seen or implied) Substitutes $\lvert XZ\rvert = 8$ and their value of $\sin\theta$ into the given formula and re-arranges for R (value of $\sin\theta$ must be valid) Correct value of R in the required form
6 (c)	Area of the circle is $\pi\left(\dfrac{16\sqrt{15}}{15}\right)^2 = \dfrac{256}{15}\pi$ Area of the triangle is $\dfrac{1}{2}(XY)(YZ)\sin\theta = \dfrac{1}{2}(6)(7)\left(\dfrac{1}{4}\sqrt{15}\right) = \dfrac{21}{4}\sqrt{15}$ So area of the shaded region is $\dfrac{256}{15}\pi - \dfrac{21}{4}\sqrt{15} = \dfrac{1}{60}\left(1024\pi - 315\sqrt{15}\right)$ **AG**	B1ft M1 A1 A1 **[4]**	Correct area of the circle ft their R (accept decimals) Correct method to find area of the triangle with values substituted and their $\sin\theta$ Correct area of the triangle (accept decimals) Subtracts the areas to obtain the given result convincingly
7 (a)	$\lim\limits_{h\to 0}\dfrac{2(x+h)-2x}{h}$ $= \lim\limits_{h\to 0}\dfrac{2x+2h-2x}{h}$ $= \lim\limits_{h\to 0} 2$ $= 2$ ***OR*** $\lim\limits_{x'\to x}\dfrac{2x'-2x}{x'-x}$ $= \lim\limits_{x'\to x}\dfrac{2(x'-x)}{(x'-x)}$ $= \lim\limits_{x'\to x} 2$ $= 2$	M1 A1 **[2]**	Considers $\dfrac{2(x+h)-2x}{h}$ or $\dfrac{2x'-2x}{x'-x}$ Complete and convincing proof with no errors and correct limiting process seen
7 (b)	$f(x) = \dfrac{px}{x^2} - \dfrac{x^{\frac{1}{2}}}{x^2} = px^{-1} - x^{-\frac{3}{2}}$ So $f'(x) == -px^{-2} + \dfrac{3}{2}x^{-\frac{5}{2}}$	M1 M1 A1 oe **[3]**	Separates into two terms with one term correct Correct method to differentiate at least one term Correct differentiation oe ISW after a correct answer seen

7 (b) ALT	$\mathrm{f}'(x) = \dfrac{x^2\left(p - \frac{1}{2}x^{-\frac{1}{2}}\right) - \left(px - \sqrt{x}\right)(2x)}{x^4}$ $\Rightarrow \mathrm{f}'(x) = px^{-2} - \frac{1}{2}x^{-\frac{5}{2}} - 2px^{-2} + 2x^{-\frac{5}{2}}$ So $\mathrm{f}'(x) == -px^{-2} + \frac{3}{2}x^{-\frac{5}{2}}$	M1 M1 A1 **[3]**	Uses product or quotient rule (formula must be correct) Correct method to differentiate at least one term in u or v Correct differentiation oe ISW after a correct answer seen
7 (c)	Gradient of curve at $x = 1$ is $-\frac{5}{2}$ So $-\frac{5}{2} = -p(1)^{-2} + \frac{3}{2}(1)^{-\frac{5}{2}}$ $\Rightarrow -\frac{5}{2} = -p + \frac{3}{2}$ $\Rightarrow p = 4$	B1 M1 A1 **[3]**	Correct gradient of the curve at $x = 1$ Forms equation using their gradient of the curve at $x = 1$ Use of 2/5 as gradient of curve at $x = 1$ is M0 Correct value of p
8	Multiplying by x^2 gives $x(4+x) > 3x^2$ $\Rightarrow 4x + x^2 > 3x^2$ $\Rightarrow 2x^2 - 4x < 0$ CVs are $x = 0$ and $x = 2$ Solution is thus $0 < x < 2$	M1 A1 A1 A1 **[4]**	Multiplies inequality by x^2 Forms correct quadratic inequality oe Correct critical values Correct solution Accept set notation
9 (a)	Incorrect statements are **B** and **D**	B1 B1 **[2]**	One mark for each incorrect statement stated If more than two incorrect statements given, withhold one B mark
9 (b)	For **B** : let the two numbers be $\sqrt{2}$ and $1-\sqrt{2}$, then their sum is 1 which is rational For **D** : let the two numbers be $\sqrt{2}$ and $\sqrt{2}$, then their product is 2 which is rational	B1 B1 B1 B1 **[4]**	Gives a counter-example for **B** (allow √2 and –√2 etc.) Correctly **shows** statement is false for their counter-example Gives a counter-example for **D** Correctly **shows** statement is false for their counter-example In each case, there is no need to justify that their starting numbers are irrational but make sure they are! Examples can also include logs, e.g. $\log_2(24) + (-\log_2(3)) = \log_2(8) = 3$

10 (a)	$\lvert\mathbf{a}\rvert = \sqrt{6^2+4^2} = 2\sqrt{13}$ so unit vector in direction of **a** is $\dfrac{6\mathbf{i}+4\mathbf{j}}{2\sqrt{13}}$	B1 B1ft **[2]**	Correct magnitude of **a** Correct unit vector in direction of **a** ft their magnitude Allow correctly written column vectors
10 (b)	$\dfrac{\lambda+2}{6} = \dfrac{1}{4}$ $\Rightarrow \lambda = -\dfrac{1}{2}$	M1 A1 **[2]**	Sets up equation with RHS = ¼ or 4 (or equivalent process) Correct value of λ
10 (c) (i)	$6\mathbf{i}+4\mathbf{j}+6\mathbf{i}+4\mathbf{j} = 12\mathbf{i}+8\mathbf{j}$	B1 **[1]**	Shows the result. Allow 2(6**i** + 4**j**) = 12**i** + 8**j**
10 (c) (ii)	Position of Q after 2 seconds $2\mathbf{i}-3\mathbf{j}+2\left(\dfrac{3}{2}\mathbf{i}+\mathbf{j}\right) = 5\mathbf{i}-\mathbf{j}$ So distance between P and Q is $\sqrt{7^2+9^2} = \sqrt{130}$	M1 A1 M1 A1 **[4]**	Attempts to find position of Q after 2 seconds Correct position of Q after 2 seconds Complete method to find distance between P and Q using their position of Q Correct exact distance cao
11 (a)	$\ln\left(\dfrac{2x+5}{x}\right) = 4$ $\dfrac{2x+5}{x} = e^4 \Rightarrow x = \ldots$ $xe^4 - 2x = 5$ $\Rightarrow x = \dfrac{5}{e^4-2}$	M1* M1(dep*) A1 **[3]**	Uses subtraction rule for logs Removes logs correctly and re-arranges for x Correct value of x (accept decimals)
11 (b)	$2^u = \dfrac{5}{e^4-2} = 0.09506\ldots$ $\Rightarrow u\log 2 = \log(0.09506\ldots)$ $\Rightarrow u = -3.395\ldots$ so $u = -3.40$ to 2 dp	M1 A1 **[2]**	Complete method to solve the equation 2^u = their (a) No marks for solving an oversimplified equation, e.g. 2^u = 1, 8, ½ etc. Correct value of u to 2 dp cao

12 (a)	$a = -2, b = 3$	B1 **[1]**	Correct values
12 (b)	$y = (0+2)^2(x-3) = -12$ So intersects y axis at (0, –12)	B1 **[1]**	Correct intersection point Condone $y = -12$ alone
12 (c) (i)	$y = (x^2+4x+4)(x-3)$ $\Rightarrow y = x^3 + x^2 - 8x - 12$ So $\frac{dy}{dx} = 3x^2 + 2x - 8$ Turning points when $\frac{dy}{dx} = 0$ $3x^2 + 2x - 8 = 0$ $\Rightarrow (3x-4)(x+2) = 0$ So minimum point occurs at $x = \frac{4}{3}$, $y = -\frac{500}{27}$	M1 A1 M1* A1 A1 **[5]**	Expands brackets of **ft their values of *a* and *b*** obtaining 4 terms with at least two correct ***OR*** complete attempt to use the product rule Correct differentiation Sets their derivative = 0 Obtains correct 3TQ Correct coordinates (accept decimals)
12 (c) (ii)	$\frac{d^2y}{dx^2} = 6x + 2$ $\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{4}{3}} = 6\left(\frac{4}{3}\right) + 2 = 10 > 0$ Since $\frac{d^2y}{dx^2}$ at $x = \frac{4}{3}$ is positive, the point is indeed a minimum	B1ft M1 A1 **[3]**	Correct second derivative ft their first derivative Substitutes the x coordinate of their minimum into their second derivative (no need to evaluate it) Shows that the second derivative is positive (no need to evaluate it, $6\left(\frac{4}{3}\right) + 2 > 0$ is OK since it is obvious) **and** conclusion, e.g. 'therefore a minimum', 'as required', 'qed', etc.
12 (d)	Values of k are $k > 0$ or $k < -\frac{500}{27}$ so in set notation this is e.g. $\left\{k \in \mathbb{R} : k > 0 \cup k < -\frac{500}{27}\right\}$	B1 **ft** B1 **ft** **[2]**	Correct range of values of k seen <u>or implied</u> ft their (c) (i) Correct values of k expressed in set notation ft their (c) (i) oe Allow $\left\{k : k > 0 \cup k < -\frac{500}{27}\right\}$

13 (a)	Gradient of $l_1 = \frac{10-0}{4--1} = 2$ So gradient of l_2 is $-\frac{1}{2}$ So equation of l_2 is $y-k=-\frac{1}{2}(x-4)$	M1 A1 A1ft A1 **[4]**	Method to find the gradient of l_1 (allow a sign error) Correct gradient of l_1 Correct gradient of l_2 ft their gradient of l_1 Correct equation of l_2 in any form
13 (b)	When $y=0,\ -k=-\frac{1}{2}(x-4) \Rightarrow x=2k+4$ Then $10=\frac{1}{2}(2k+4)(10)$ $\Rightarrow 2=2k+4$ $\Rightarrow 2k=-2$ $\Rightarrow k=-1$ (as required)	M1* M1(dep*) A1 **[3]**	Substitutes $y = 0$ into their (a) and attempts to re-arrange for x Sets up a correct equation using their $2k + 4$ Complete and convincing proof with no errors seen
13 (c)	$l_1: y=2x+2$, $l_2: y+1=-\frac{1}{2}(x-4)$ Substituting gives $2x+3=-\frac{1}{2}(x-4)$ $\Rightarrow 4x+6=-x+4$ $\Rightarrow 5x=-2$ $\Rightarrow x=-\frac{2}{5}$ then $y=2\left(-\frac{2}{5}\right)+2=\frac{6}{5}$ So coordinates of intersection are $\left(-\frac{2}{5},\frac{6}{5}\right)$	B1 M1 A1 A1 **[4]**	Correct equation of the line l_1 Eliminates one of the variables from their l_1 and their l_2 **which has *k* = –1 correctly substituted** Correct x coordinate Correct y coordinate

14	$f(x)=\int 2\sqrt{x}\,dx=\dfrac{4x^{\frac{3}{2}}}{3}+c$	M1*	Attempts to integrate to find f(x). Must include a constant of integration
	Passes through (4,1) so $1=\dfrac{4}{3}(4)^{\frac{3}{2}}+c\Rightarrow c=-\dfrac{29}{3}$	M1**(dep*)	Substitutes initial condition into their integral to find the constant
	So $y=\dfrac{4}{3}x^{\frac{3}{2}}-\dfrac{29}{3}$	A1	Correct y in terms of x (can be implied)
	Gradient of curve when $x = 9$ is $2\sqrt{9}=6$	B1	Correct gradient of curve when $x = 9$
	y coordinate when $x = 9$ is $\dfrac{79}{3}$	B1ft	Correct y coordinate of curve when $x = 9$ ft their y in terms of x
	Hence equation of tangent is $y-\dfrac{79}{3}=6(x-9)$	M1(dep**)	Attempts to find the equation of the tangent using their gradient, 9 and their y coordinate
	$\Rightarrow 3y-79=18x-162$ $\Rightarrow 18x-3y-83=0$	A1 **[7]**	Correct equation of the tangent in the required form Accept equivalent answers that are also in this form