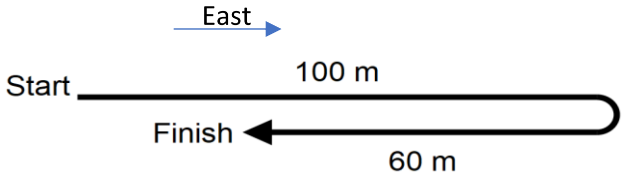
A LEVEL MATHS - MECHANICS REVISION NOTES

1 KINEMATICS

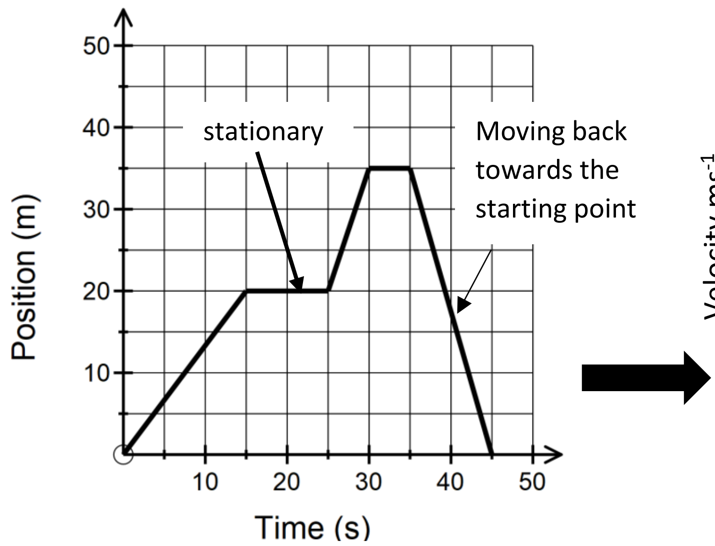
- **Distance** - a scalar quantity with no direction
  = 160 m
- **Displacement** - a vector quantity – measured from the starting position
  = 40 m (East of starting point)
- **Position** - a vector quantity – distance from a fixed origin

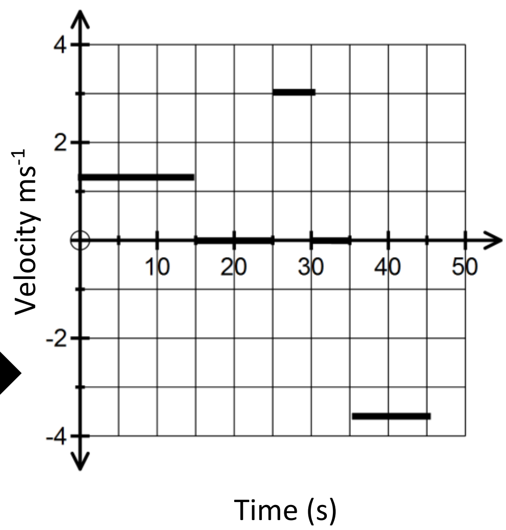


AVERAGE SPEED = $\frac{Total\ Distance}{Total\ Time}$ **AVERAGE VELOCITY**= $\frac{Displacement}{Time\ taken}$

USING Position-Time and Velocity-Time GRAPHS

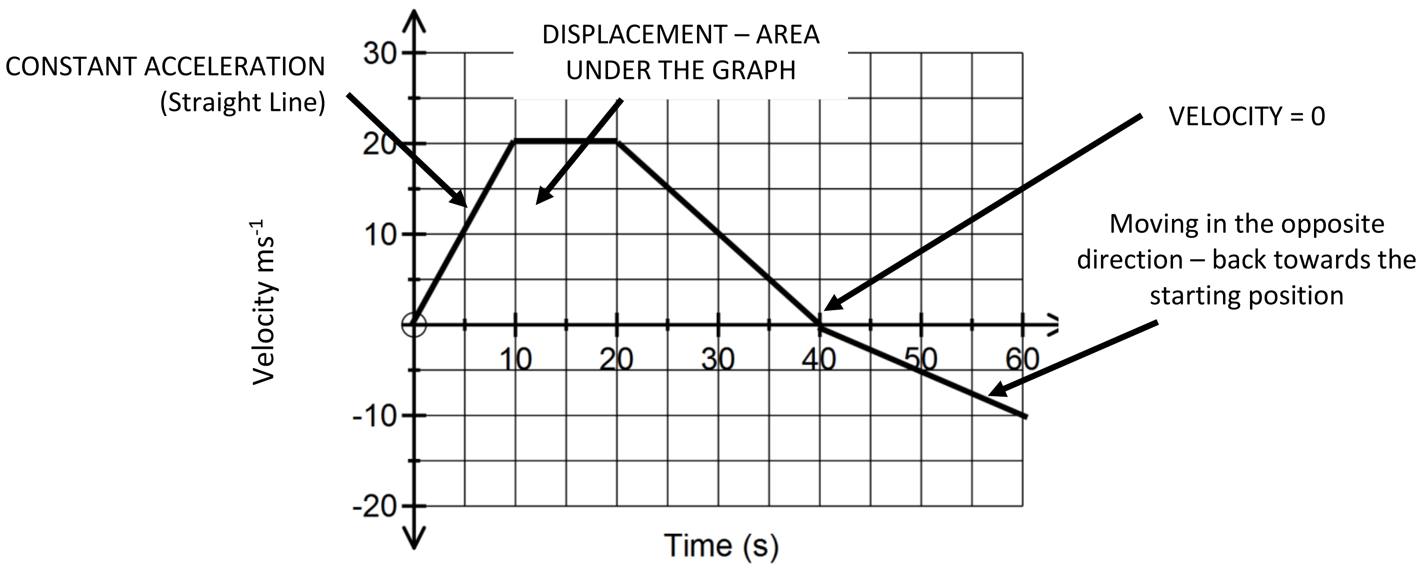
Position- time graph **Velocity – time graph**





VELOCITY TIME GRAPH

Gradient = acceleration



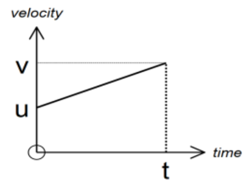
EQUATIONS FOR CONSTANT ACCELERATION -

s: displacement (m)   u : initial velocity (ms^{-1})   v : final velocity (ms^{-1})   a : acceleration (ms^{-2})

t = time (s)

$$v = u + at \qquad v^2 = u^2 + 2as$$

$$s = \tfrac{1}{2}(u + v)t \qquad s = ut + \tfrac{1}{2}at^2 \qquad s = vt - \tfrac{1}{2}at^2$$



- Acceleration due to gravity is **9.8 ms^{-2}** (unless given in the question)
- Negative Acceleration means retardation/deceleration
- You may need to show how the equations can be derived from the graph

A car starts from rest and reaches a speed of 15 ms^{-1} after travelling 25m with constant acceleration. Assuming the acceleration remains constant, how much further will the car travel the next 4 seconds?

u = 0 ms^{-1}
v = 15 ms^{-1}
s = 25 m

$v^2 = u^2 + 2as \quad 15^2 = 2a \times 25$
$a = 4.5\ ms^{-2}$

u = 15 ms^{-1}
t = 4
a = 4.5 $s = ut + \tfrac{1}{2}at^2$
$s = 15 \times 4 + \tfrac{1}{2} \times 4.5 \times 16$
$= 96$ m

A ball is thrown vertically upwards with a speed of 12 ms^{-1} from a height of 1.5 m. Calculate the maximum height reached by the ball.

u = 12 ms^{-1}
a = -9.8 ms^{-2}
At maximum height v = 0
$v^2 = u^2 + 2as$
$0 = 144 - 2 \times 9.8 \times s$
s = 7.35 m

Maximum height = 1.5 + 7.35
= 8.85 m

2 FORCES and ASSUMPTIONS

KEY FORCES

W : weight (mg = mass × 9.8)
R : reaction (normal reaction – at right angles to the point of contact)
F : friction (acts in a direction opposite to that in which the object is moving or is on the point of moving)
T : Tension

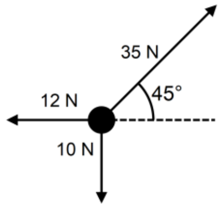
ASSUMPTIONS

- Objects are modelled as masses concentrated at a single point so no rotational forces.
- Strings are inextensible (inelastic) so any stretch can be disregarded
- Strings and rods are light (no mass) so weight can be disregarded
- Pulleys are smooth so no frictional force at the pully needs to be considered.

RESOLVING FORCES

Calculate the resultant force acting on the particle giving you answer in the form ai + bj

Working in i (horizontal)
R**i** = 35cos45° - 12
= 12.7 N



Working in j (vertical)
R**j** = 35sin45° - 10
= 14.7 N

R = 12.7 **i** + 14.7 **j**

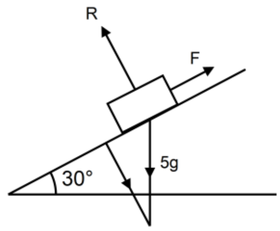
The forces shown in the diagram act on particle A of mass 0.8 kg. Calculate the magnitude of the acceleration of the particle
Resultant force = 12.7**i** + 14.7**j**
Force = mass × acceleration
0.8**a** = 12.7**i** + 14.7**j**
a = 15.875**i** + 18.375**j**

$|\mathbf{a}| = \sqrt{15.875^2 + 18.375^2}$

$|\mathbf{a}|$ = 24.3 ms^{-2} (3 s.f.)

If the system is in **Equilibrium**, then resultant force = 0
Take care with objects on slope – always draw a diagram showing all the forces

A box of mass 5 kg rests on a slope inclined at an angle of 30° to the horizontal
Calculate the normal reaction and the friction



Resolving perpendicular to the slope R = 5g cos 30°
= 42.4 N (3 s.f)

Resolving parallel to the slope F = 5g sin 30°
= 24.5 N(3 s.f)

COEFFICIENT OF FRICTION

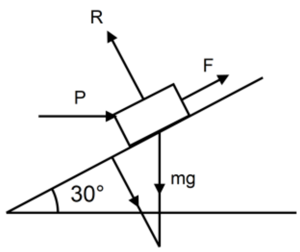
The maximum or limiting value of friction F_{max} is given by

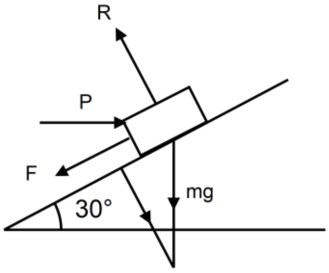
$F_{max} = \mu R$ R is the normal reaction and μ is the **coefficient of friction**

If a force is acting on the object but the object remains at rest then F < μR
When the object is moving the frictional force is constant (F_{max})

For questions looking at the minimum and maximum force needed to move a block on a rough slope look at the magnitude of force P

When the block is on the verge of sliding down the slope
Friction is 'acting up the slope'



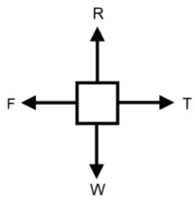


When the block is on the verge of sliding up the slope
Friction is 'acting down the slope'

3 NEWTONS LAWS

1st LAW : Every object remains at rest or moves with constant velocity unless an external force is applied

The system is in **EQUILIBRIUM**



T = F
R = W

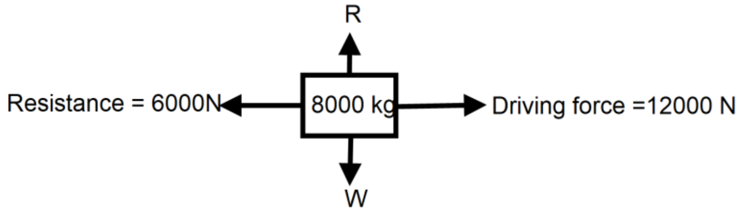
2nd LAW : The resultant force acting on an object is equal to the acceleration of that body times its mass.

F = ma

3rd LAW : If an object A exerts a force on object B, then object B must exert a force of equal magnitude and opposite direction back on object A.

Calculate the acceleration of the object
Resultant force = 12000 – 6000
= 6000 N

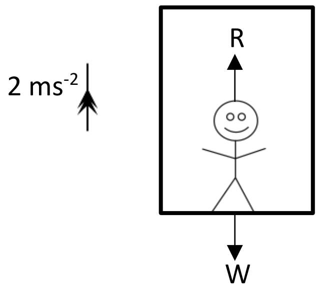
6000 = 8000a   a = 0.75 ms^{-2}



A man of mass 80 kg stands in a lift
Calculate the normal reaction of the lift floor on the man if

a) The lift is moving downwards with constant velocity
Constant velocity so R = W
R = 80g N
= 784 N

b) The lift is moving upwards with acceleration of 2 ms^{-2}
Upwards movement so R > W   R – 80g = 80×2
R = 944 N

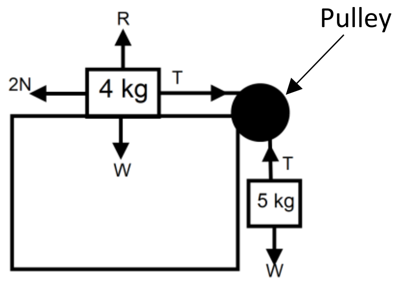


Two masses are connected by a light string passing over a smooth pulley as shown below.
Calculate the acceleration of the 4 kg block when released from rest.

5 kg Block : 5g – T = 5a
4kg Block :   T – 2 = 4a

Solving simultaneously   5g – 2 = 9a

a = 5.22 ms^{-2}



Forces $F_1 = 2\mathbf{i} + \mathbf{j}$, $F_2 = -3\mathbf{i} + 4\mathbf{j}$ and $F_3 = 4\mathbf{i} - 6\mathbf{j}$ act on a particle with mass 10 kg. Find the magnitude of acceleration of the particle

Resultant force = $F_1 + F_2 + F_3 = (2\mathbf{i} + \mathbf{j}) + (-3\mathbf{i} + 4\mathbf{j}) + (4\mathbf{i} - 6\mathbf{j})$
$= 3\mathbf{i} - \mathbf{j}$

F = ma

$3\mathbf{i} - \mathbf{j} = 10a$ $a = 0.3\mathbf{i} - 0.1\mathbf{j}$ $|a| = \sqrt{0.3^2 + (-0.1)^2}$ a = 0.316 ms^{-2}

4 VARIABLE ACCELERATION (r : position)

r = f(t)	↓ DIFFERENTIATING	**Displacement r (m)**	↑ INTEGRATING - Don't forget to include + c	r = $\int v \ dt$
v = $\frac{dr}{dt}$		**Velocity v (ms^{-1})**		v = $\int a \ dt$
a = $\frac{dv}{dt}$ = $\frac{d^2r}{dt^2}$		**Acceleration a (ms^{-2})**		a = f(t)

Remember

- Area under a velocity time graph = displacement
- Gradient at a point on position/time graph = velocity
- Gradient at a point on velocity/time graph = acceleration

The acceleration of a particle (in ms^{-2}) at time t seconds is given by a = 12 – 2t.
The particle has an initial velocity of 3 ms^{-1} when it starts at the origin.

a) Find the velocity of the particle after t seconds

v = $\int 12 - 2t \, dt$

v = 12t – t^2 + c   t = 0 v = 3   c = 3

v = 12t – t^2 + 3

b) Find the position of the particle after t seconds

r = $\int 12t - t^2 + 3 \, dt$

= $6t^2 - \frac{t^3}{3} + 3t + c$

r = 0   t = 0   r = $6t^2 - \frac{t^3}{3} + 3t$

A train moves between 2 stations, stopping at both of them
Its speed at t seconds is modelled by V = $\frac{1}{5000}$t(1200 – t) (ms^{-1})
Find the distance between the 2 stations

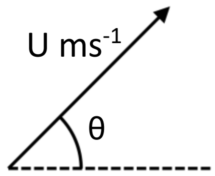
At the stations v = 0 $\frac{1}{5000}$t(1200 – t) = 0   t = 0 t = 1200

Distance = $\int_0^{1200} \frac{1}{5000} \mathrm{t}(1200 - \mathrm{t}) dt \;=\; \frac{1}{5000}[600t^2 - \frac{t^3}{3} + c]$

= 57600 m
= 57.6 km

5 PROJECTILES

Initial Velocity : u = $U\cos\theta$ **i** + $U\sin\theta$ **j**



Acceleration : a = -g **j**

Velocity after t seconds:

$v = U\cos\theta\ \mathbf{i} + (U\sin\theta - gt)\ \mathbf{j}$

Particle moving in a horizontal direction (reaches **maximum height**)
when **j** component = 0 $\quad U\sin\theta - gt = 0$

Displacement after t seconds (r)

$R = Ut\cos\theta\ \mathbf{i} + (Ut\sin\theta\ j - \frac{g}{2}t^2)\mathbf{j}$

If launched from the ground the particle will return to the ground
when $r_j = 0$ $\quad Ut\sin\theta - \frac{g}{2}t^2 = 0$

Solve the equation to find the values of t. Substitute 't' into the **i** component of r to calculate **the range**

A shot putter releases a shot at a height of 2.5m, with speed 10 ms^{-1} at an angle of 50° to the horizontal. Calculate the horizontal distance from the thrower to where the shot lands.

u = $10\cos50°$ **i** + $10\sin50°$**j**

a = -9.8 ms^{-2}**j**

s = $10t\cos50°$ **i** + ($10t\sin50° - 4.9t^2$) **j**

Starting height = 2.5 m so hits the ground when $s_j = -2.5$

$10t\sin50°\mathbf{j} - 4.9t^2 = -2.5$

(t = -0.277) or t = 1.84 Horizontal distance travelled when t = 1.84

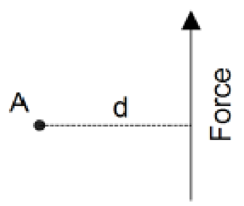
$= 10 \times 1.84 \cos 50°$

= 11.8 m (3 s.f.)

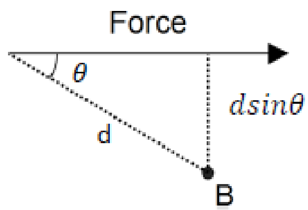
6 MOMENTS

The moment of a force about point A is
moment = Force × d (Nm) (anticlockwise)

d is the perpendicular distance of the line of action of the force from A



The moment of a force about point B is
moment = Force × dsin θ (Nm)
(clockwise)



The resultant moment is the difference between the sum of the clockwise moments and sum of the anticlockwise moments (in the direction of the larger sum)

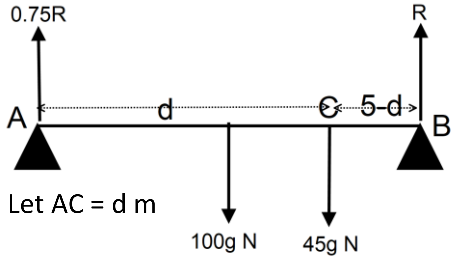
Uniform Lamina - (usually rectangular) – has same density throughout – Centre of mass through which the objects weight acts is the centre of the rectangle

Uniform Rod centre of mass is at the midpoint of the rod

Equilibrium
If an object is in equilibrium the **resultant force is zero** and the **total moment of all the forces is zero**
To solve problems

- Draw a diagram showing all the forces
- By taking moments about a point you can ignore the forces acting at that point (d = 0)
- Resolve the forces horizontally, vertically

A uniform bridge across a steam is 5m long and has a mass of 100 kg. It is supported at the ends A and B. A child of mass 45 kg is standing on the bridge at point C. Given that the magnitude of the force exerted by the support at A is three-quarters of the magnitude of the force exerted by the support at B calculate the magnitude of the force exerted at support A and the distance AC



Let AC = d m

Resolving vertically : $1.75R = 145g$

$R = 82.9g$ N (3 s.f.)

Force at A = $0.75R$

$= 62.1g$ N

Taking moments about A: $(2.5\times100) + (45\times d) = 5\times82.9$

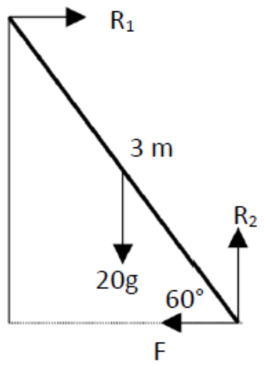
$250 + 45d = 414.5$

$d = 3.66$ m

NOT ON ALL EXAM BOARDS

A uniform ladder of length 3 m, and mass 20 kg, leans against a smooth, vertical wall, so that the angle between the horizontal ground and the ladder is 60°. Find the magnitude of the friction and the normal reaction forces that act on the ladder if it is in equilibrium

Step 1: **Draw a diagram showing all of the forces**



Step 2 : **Resolving Vertically**

$R_2 = 20g \quad R_2 = 196$ N

Step 3 : **Taking moments at the base**

$20g\times1.5\cos60° = R_1\times3\sin60°$

$R_1 = 56.6$ N

Step 4 : **Resolving horizontally**

$F = R_1 = 56.6$ N