KS5 "Full Coverage": Binomial Expansion (Year 2)

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Determine the binomial expansion of $(1 + kx)^n$ for negative or fractional k.

[OCR C4 June 2014 Q3i] Find the first three terms in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ in ascending powers of x , where $|x| < \frac{1}{2}$.

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Question 2

Categorisation: Determine the binomial expansion of $(a + kx)^n$

[Edexcel C4 Jan 2011 Q5a] Use the binomial theorem to expand

$(2 - 3x)^{-2}$, $|x| < \frac{2}{3}$,

in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

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Question 3

Categorisation: As above, but for fractional n.

[Edexcel A2 Specimen Papers P1 Q2a] Show that the binomial expansion of $(4+5x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2 is

$$2+\frac{5}{4}x+kx^2$$

giving the value of the constant k as a simplified fraction.

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Question 4

Categorisation: Use a Binomial expansion to determine an approximation for a square root.

[Edexcel A2 Specimen Papers P1 Q2bi Edited]

It can be shown that the binomial expansion of $(4+5x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2 is

$$2+\frac{5}{4}x-\frac{25}{64}x^2$$

Use this expansion with $x=\frac{1}{10}$, to find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

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Question 5

Categorisation: Understand that Binomial expansions are only valid for particular ranges of values for x.

[Edexcel A2 Specimen Papers P1 Q2bii Edited] (Continued from above)

Explain why substituting $x = \frac{1}{10}$ into this binomial expansion leads to a valid approximation.

Question 6

Categorisation: Understand that $\sqrt{\ldots}$ can be written as $(\ldots)^{\frac{1}{2}}$ in order to a Binomial expansion.

[Edexcel C4 June 2018 Q1a] Find the binomial series expansion of

$\sqrt{4 - 9x}$, $|x| < \frac{4}{9}$

in ascending powers of x , up to and including the term in x^2 Give each coefficient in its simplest form.

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Question 7

Categorisation: Use substitution to determine square/cube roots, but more difficult ones where a direct comparison would lead to an invalid value of x.

[Edexcel C4 June 2013(R) Q4b Edited] The binomial expansion of $\sqrt[3]{8-9x}$ up to and including the term in x^3 is written below.

$$\sqrt[3]{8-9x} = 2 - \frac{3}{4}x - \frac{9}{32}x^2 - \frac{45}{256}x^3, \qquad |x| < \frac{8}{9}$$

Use this expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

Question 8

Categorisation: As above.

[Edexcel C4 June 2018 Q1b Edited]

It can be shown that

$$\sqrt{4-9x} \approx 2 - \frac{9}{4}x - \frac{81}{64}x^2, \qquad |x| < \frac{4}{9}$$

Use this expansion, with a suitable value of x, to find an approximate value for $\sqrt{310}$ Give your answer to 3 decimal places.

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Question 9

Categorisation: Rewrite more complication expressions (involving fractions) as a product of two Binomial expansions.

[OCR C4 June 2012 Q3ii Edited]

It can be shown that $\frac{1+x^2}{\sqrt{1+4x}} \approx 1 - 2x + 7x^2 - 22x^3$

State the set of values of x for which this expansion is valid.

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Question 10

Categorisation: As per Q5, but involving further problem solving.

[OCR C4 June 2016 Q7]

Given that the binomial expansion of $(1 + kx)^n$ is $1 - 6x + 30x^2 + \cdots$. find the values of n and k .

State the set of values of x for which this expansion is valid.

$n =$ ……………………

$k =$ ……………………

$|x| <$ ……………………

Question 11

Categorisation: Multiply a Binomial expansion by a further bracket.

[Edexcel C4 June 2014(R) Q1b Edited] Given that

$$\frac{1}{\sqrt{9-10x}} = \frac{1}{3} + \frac{5}{27}x + \frac{25}{162}x^2 + \cdots$$

where $|x| < \frac{9}{10}$, find the expansion of

$$\frac{3+x}{\sqrt{9-10x}}$$

$|x| < \frac{9}{10}$, in ascending powers of x , up to and including the term in x^2 .

Give each coefficient as a simplified fraction.

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Question 12

Categorisation: As per Q9.

[OCR C4 June 2011 Q6]

Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1+ax}{4-x}}$$

giving your answer in terms of a .

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Question 13

Categorisation: As above.

[Edexcel C4 June 2013 Q2a Edited]

Find the binomial expansion of

$$\sqrt{\frac{1+x}{1-x}}$$

where $|x| < 1$ up to and including the term in x^2 .

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Question 14

Categorisation: Use partial fractions to find a Binomial expansion.

[Edexcel C4 June 2010 Q5b Edited]

It is given that:

$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv 2 - \frac{1}{x-1} + \frac{4}{x+2}$$

Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x , as far as the term in x^2 . Give each coefficient as a simplified fraction.

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Question 15

Categorisation: Determine the Binomial expansion of $\left(a + bx^k\right)^n$, i.e. involving a more general power of x within the bracket.

[Edexcel C4 June 2011 Q2]

$$f(x) = \frac{1}{\sqrt{9 + 4x^2}}$$

where $|x| < \frac{3}{2}$.

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x. Give each coefficient as a simplified fraction.

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Answers

Question 1

$1 + (-\frac{1}{2})(-2x) + (-\frac{1}{2})(\frac{-3}{2})\frac{(\pm 2x)^2}{2!}[+ ...]$	B1 B1
$1 + x + \frac{3}{2}x^2$ oe	B1

Question 2

(a)	$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1
	$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ ...$ $= 1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ ...$	M1 A1
	$(2-3x)^{-2} = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ ...$	M1 A1

Question 3

$(4+5x)^{\frac{1}{2}} = (4)^{\frac{1}{2}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = 2\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	B1	1.1b
$= \{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5x}{4}\right)^2+\ ...\right]$	M1	1.1b
	A1ft	1.1b
$= 2+\frac{5}{4}x-\frac{25}{64}x^2+\ ...$	A1	2.1

Question 4

$\left\{x=\frac{1}{10}\Rightarrow\right\}(4+5(0.1))^{\frac{1}{2}}$	M1	1.1b
$= \sqrt{4.5} = \frac{3}{2}\sqrt{2}$ or $\frac{3}{\sqrt{2}}$		
$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ ...\ \{=2.121...\}$ $\Rightarrow \frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\sqrt{2}}=\frac{543}{256} \Rightarrow \sqrt{2}= ...$	M1	3.1a
So, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	A1	1.1b

Question 5

$x=\frac{1}{10}$ satisfies $\lvert x\rvert<\frac{4}{5}$ (o.e.), so the approximation is valid.	B1	2.3

Question 6

$\sqrt{(4-9x)} = (4-9x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	B1
$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$		
$=2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$	see notes	
$= 2-\frac{9}{4}x; -\frac{81}{64}x^2+\ldots$	isw	A1; A1

Question 7

$\left\{\sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)},\right\}$ so $x = 0.1$	Writes down or uses $x = 0.1$	B1
When $x = 0.1$, $\sqrt[3]{(8-9x)} \approx 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3+\ldots$ $= 2-0.075-0.0028125-0.00017578125$ $= 1.922011719$		M1
So, $\sqrt[3]{7100} = 19.220117919\ldots = \underline{19.2201}$ (4 dp)	19.2201 **cso**	A1 **cao**

Question 8

$\sqrt{310} = 10\sqrt{3.1} = 10\sqrt{(4-9(0.1))}$, so $x = 0.1$	E.g. For $10\sqrt{3.1}$ (can be implied by later working) **and** $x = 0.1$ (or uses $x = 0.1$) **Note:** $\sqrt{(100)(3.1)}$ by itself is B0	B1
When $x = 0.1$ $\sqrt{(4-9x)} \approx 2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2+\ldots$	Substitutes their x, where $\lvert x\rvert < \frac{4}{9}$ into all three terms of their binomial expansion	M1
$= 2-0.225-0.01265625 = 1.76234375$		
So, $\sqrt{310} \approx 17.6234375 = \underline{17.623}$ (3 dp)	17.623 **cao**	A1 **cao**
Note: the calculator value of $\sqrt{310}$ is 17.60681686... which is 17.607 to 3 decimal places		

Question 9

$\lvert x\rvert < \frac{1}{4}; -\frac{1}{4} < x < \frac{1}{4}; \{-\frac{1}{4} < x\ ,\ x < \frac{1}{4}\}$ no equality	B1

Question 10

$nk = -6$ soi	**B1**
$\frac{n(n-1)k^2}{2!} = 30$ soi	**B1**
substitution of $n = \pm\frac{6}{k}$ or $k = \pm\frac{6}{n}$ or $k = \pm\sqrt{\frac{60}{n(n-1)}}$ oe to eliminate one variable from their equations	**M1**
$n = -1.5$ oe	**A1**
$k = 4$	**A1**
expansion is valid for $\lvert x\rvert < \frac{1}{4}$ or $-\frac{1}{4} < x < \frac{1}{4}$ isw	**B1FT**

Question 11

$1+\frac{8}{9}x+\frac{35}{54}x^2$

(b)	$\frac{3+x}{\sqrt{(9-10x)}}=(3+x)(9-10x)^{-\frac{1}{2}}$		
	$=(3+x)\left(\frac{1}{3}+\frac{5}{27}x+\left\{\frac{25}{162}x^2+\right\}...\right)$	*Can be implied by later work* **See notes**	M1
	$=1+\frac{5}{9}x+\frac{25}{54}x^2+\frac{1}{3}x+\frac{5}{27}x^2+...$	Multiplies out to give exactly one constant term, exactly 2 terms in x and exactly 2 terms in x^2. Ignore terms in x^3. Can be implied.	M1
	$=1+\frac{8}{9}x+\frac{35}{54}x^2+...$		A1

Question 12

$-\frac{1}{16}a^2+\frac{1}{32}a+\frac{3}{256}$

$(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}ax$ .... $+\frac{\frac{1}{2}\cdot\frac{-1}{2}}{2}(ax)^2$ B1,B1 N.B. third term $=-\frac{1}{8}a^2x^2$

Change $(4-x)^{-\frac{1}{2}}$ into $k\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$, where k is likely to be $\frac{1}{2}$/2/4/−2, & work out expansion of $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\frac{1}{8}x$ .... $+\frac{\frac{-1}{2}\cdot\frac{-3}{2}}{2}\left(\frac{(-)x}{4}\right)^2$ B1,B1 N.B. third term $=\frac{3}{128}x^2$

<u>OR</u> Change $\{4-x\}^{\frac{1}{2}}$ into $l\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$, where l is likely to be $\frac{1}{2}$/2/4/−2,& work out expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{\frac{1}{2}}=1-\frac{1}{8}x-\frac{1}{128}x^2$ B1 (for all 3 terms simplified)

$k=\frac{1}{2}$ (with possibility of M1 + A1 + A1 to follow) B1 $l=2$ (<u>with no further marks available</u>)

Multiply $(1+ax)^{\frac{1}{2}}$ by $(4-x)^{-\frac{1}{2}}$ or $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1 Ignore irrelevant products

The required three terms (with/without x^2) <u>identified as</u>

$-\frac{1}{16}a^2+\frac{1}{32}a+\frac{3}{256}$ or $\frac{-16a^2+8a+3}{256}$ AEF ISW A1+A1 **8** A1 for one correct term + A1 for other two

Question 13

$1+x+\frac{1}{2}x^2$

Question 14

(b)	$\frac{2x^2+5x-10}{(x-1)(x+2)}=2+(1-x)^{-1}+2\left(1+\frac{x}{2}\right)^{-1}$		M1
	$(1-x)^{-1}=1+x+x^2+ ...$		B1
	$\left(1+\frac{x}{2}\right)^{-1}=1-\frac{x}{2}+\frac{x^2}{4}+ ...$		B1
	$\frac{2x^2+5x-10}{(x-1)(x+2)}=(2+1+2)+(1-1)x+\left(1+\frac{1}{2}\right)x^2+ ...$		M1
	$=5+ ...$	ft their $A-B+\frac{1}{2}C$	A1 ft
	$= ... +\frac{3}{2}x^2+ ...$	$0x$ stated or implied	A1 A1

Question 15

$\mathrm{f}(x) = (\ \dots + \dots)^{-\frac{1}{2}}$		M1
$= 9^{-\frac{1}{2}}(\ \dots + \dots)^{\dots}$	3^{-1}, $\frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$	B1
$\left(1+kx^2\right)^n = 1 + nkx^2 + \dots$	n not a natural number, $k \neq 1$	M1
$\left(1+kx^2\right)^{-\frac{1}{2}} = \dots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(kx^2\right)^2$	ft their $k \neq 1$	A1 ft
$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$		A1
$\mathrm{f}(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$		A1