GOLD

The equation $x^3 - 8x^2 + cx + d = 0$ where $c$ and $d$ are real numbers, has roots $\alpha, \beta, \gamma$.

When plotted on an Argand diagram, the triangle with vertices at $\alpha, \beta, \gamma$ has an area of 8.

Given $\alpha = 2$, find the values of $c$ and $d$.

Fully justify your solution.

**[5 marks]**

SILVER

The cubic equation $z^3 - 3z^2 + z + 5 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where $p$, $q$ and $r$ are integers to be found.

**(5)**

BRONZE

The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients $p$ and $q$ are real, has a complex root $\sqrt{5} - \mathrm{i}$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of $p$ and $q$. *(2 marks)*