1. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball of mass 0.5 kg is moving with velocity (10**i** + 24**j**) ms^{-1} when it is struck by a bat. Immediately after the impact the ball is moving with velocity 20**i** ms^{-1}.

Find

(a) the magnitude of the impulse of the bat on the ball,

(4)

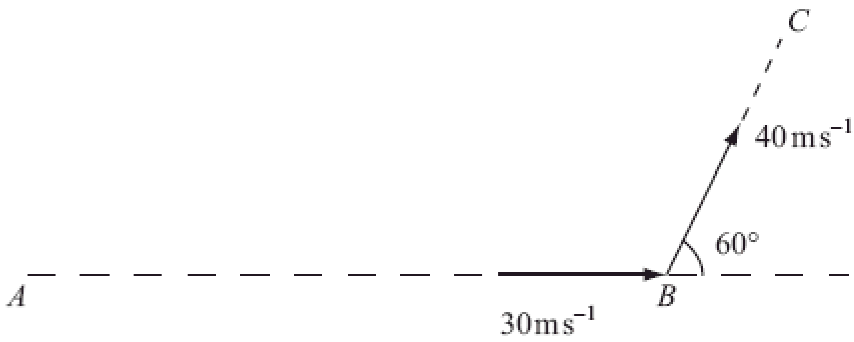
(b) the size of the angle between the vector *i* and the impulse exerted by the bat on the ball,

(2)

(c) the kinetic energy lost by the ball in the impact.

(3)
(Total 9 marks)

2.



The points A, B and C lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg. Immediately before being struck, the ball is moving along the horizontal line AB with speed 30 m s^{-1}. Immediately after being struck, the ball moves along the horizontal line BC with speed 40 m s^{-1}. The line BC makes an angle of 60° with the original direction of motion AB, as shown in the diagram above.

Find, to 3 significant figures,

(i) the magnitude of the impulse given to the ball,

(ii) the size of the angle that the direction of this impulse makes with the original direction of motion AB.

(Total 8 marks)

3. A particle of mass 0.25 kg is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ m s^{-1} when it receives the impulse $(5\mathbf{i} - 3\mathbf{j})$ N s.

Find the speed of the particle immediately after the impulse.

(Total 5 marks)

4. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}$ $(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$.

(4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall.

(4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall.

(4)
(Total 17 marks)

5. A particle P of mass 0.4 kg is moving so that its position vector $\mathbf{r}$ metres at time t seconds is given by

$$\mathbf{r} = (t^2 + 4t)\mathbf{i} + (3t - t^3)\mathbf{j}.$$

(a) Calculate the speed of P when $t = 3$.

(5)

When $t = 3$, the particle P is given an impulse $(8\mathbf{i} - 12\mathbf{j})$ N s.

(b) Find the velocity of P immediately after the impulse.

(3)
(Total 8 marks)

6. A tennis ball of mass 0.2 kg is moving with velocity $(-10\mathbf{i})$ m s^{-1} when it is struck by a tennis racket. Immediately after being struck, the ball has velocity (15**i**+ 15**j**) m s^{-1}. Find

(a) the magnitude of the impulse exerted by the racket on the ball,

(4)

(b) the angle, to the nearest degree, between the vector **i** and the impulse exerted by the racket,

(2)

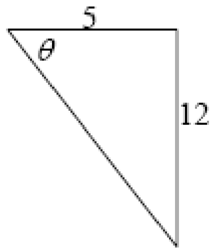
(c) the kinetic energy gained by the ball as a result of being struck.

(2)
(Total 8 marks)

1. (a)

$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$		
$= 0.5 \times 20\mathbf{i} - 0.5(10\mathbf{i} + 24\mathbf{j})$	M1	
$= 5\mathbf{i} - 12\mathbf{j}$	A1	
$\lvert 5\mathbf{i} - 12\mathbf{j} \rvert = 13$ Ns	M1 A1	4

(b)



$\tan\theta = \frac{12}{5}$	M1	
$\theta = 67.38$		
$\theta = 67.4°$	A1	2

(c)

$\text{K.E.lost} = \frac{1}{2} \times 0.5(10^2 + 24^2) - \frac{1}{2} \times 0.5 \times 20^2$	M1 A1	
$= 69$ J	A1	3

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2. (i)

$I \uparrow = 0.25 \times 40\sin 60 = 5\sqrt{3}$ (8.66)	one component	M1	
$I \leftarrow = 0.25(-20 + 30) = 2.5$	both	A1	
$\lvert I \rvert = \sqrt{75 + 6.25} = 9.01$ (Ns)		M1 A1	4

<u>Alternative</u>

Use of $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$	M1	
$30^2 + 40^2 - 2 \times 30 \times 40\cos 60°$ (= 1300)	M1 A1	
$I = 0.25\sqrt{1300} = 9.01$ N s (3 s.f.)	A1	

<u>2nd Alternative</u>

$\mathbf{u} = 30\mathbf{i}$, $\mathbf{v} = 40\cos 60\mathbf{i} + 40\sin 60\mathbf{j} = 20\mathbf{i} + 20\sqrt{3}\,\mathbf{j}$	M1	
$I = \frac{1}{4}\left(-10\mathbf{i} + 20\sqrt{3}\mathbf{j}\right) = -2.5\mathbf{i} + 5\sqrt{3}\mathbf{j}$	A1 etc	

(ii) $\frac{\sin\theta}{40} = \frac{\sin 60°}{\sqrt{1300}}$

$\theta = 106°$ (3 s.f.) M1 A1

or $\tan\theta = \pm\frac{5\sqrt{3}}{2.5}$ oee $\theta = 106°$ M1 A1 4

[8]

3. **I** $= m\mathbf{v} - m\mathbf{u}$

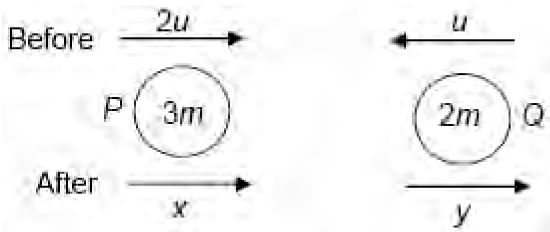
$5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$ M1A1

$\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$ A1

$|\mathbf{v}| = \sqrt{23^2 + 5^2} = 23.5$ M1A1

[5]

4. (a)



Correct use of NEL M1 *
$y - x = e(2u + u)$ o.e. A1

CLM (→): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$) B1
Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$) d * M1
Hence, speed of Q = $\underline{\frac{1}{5}(9e+4)u}$ **AG** A1 cso 5

(b) $x = y - 3eu = \frac{1}{5}(9e+4)u - 3eu$ M1 *

Hence, speed $P = \frac{1}{5}(4-6e)u = \frac{2u}{5}(2-3e)$ o.e. A1

$x = \frac{1}{2}u = \frac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e d * M1

gives, $e = \frac{3}{12} \Rightarrow$ $\underline{\mathbf{e = \frac{1}{4}}}$ **AG** A1

Or

Using NEL correctly with given speeds of P and Q M1 *
$3eu = \frac{1}{5}(9e+4)u - \frac{1}{2}u$ A1
$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e d * M1
$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$. A1 4

(c)	Time taken by Q from A to the wall = $\frac{d}{\underline{y}} = \left\{\frac{4d}{5u}\right\}$	M1[+]	
	Distance moved by P in this time = $\frac{u}{2} \times \frac{d}{y} \left(= \frac{u}{2}\left(\frac{4d}{5u}\right) = \frac{2}{5}d\right)$	A1	
	Distance of P from wall = $d - \times\left(\frac{d}{y}\right); = d - \frac{2}{5}d = \underline{\frac{3}{5}d}$ **AG**	d[+]M1;	
		A1 cso	
	or		
	Ratio speed P:speed Q = x:y = $\frac{1}{2}u : \frac{1}{5}(\frac{9}{4}+4)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$	M1[+]	
	So if Q moves a distance d, P will move a distance $\frac{2}{5}d$	A1	
	Distance of P from wall = $d - \frac{2}{5}d; = \underline{\frac{3}{5}d}$ **AG** cso	d[+]M1;A1	4
(d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft	
	Time for P, $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y	B1ft	
	Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$	M1	
	gives, $2(\frac{3d}{5} - x) = 4x \Rightarrow \frac{3d}{5} - x = 2x, 3x = \frac{3d}{5} \Rightarrow x = \underline{\frac{1}{5}d}$	A1 cao	
	or		
	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft	
	speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y	B1 ft	
	Distance of B from wall = $\frac{1}{3} \times \frac{3d}{5}; = \frac{d}{5}$ their $\frac{1}{2+1}$	M1; A1	4

2nd or

After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$

Time from wall to 2nd collision = $\frac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y B1ft

Distance of B from

wall = (their speed)x(their time) = $\frac{u}{4} \times \frac{4d}{5u}; = \underline{\frac{1}{5}d}$ M1; A1 4

[17]

5. (a) $\dot{\mathbf{r}} = (2t + 4)\mathbf{i} + (3 - 3t^2)\mathbf{j}$ M1 A1

$\dot{\mathbf{r}}_3 = 10\mathbf{i} - 24\mathbf{j}$ substituting $t = 3$ M1

$|\dot{\mathbf{r}}_3| = \sqrt{(10^2 + 24^2)} = 26$ (m s^{-1}) M1 A1 5

(b) $0.4(\mathbf{v} - (10\mathbf{i} - 0\ 24\mathbf{j}) = 8\mathbf{i} - 12\mathbf{j}$ ft their $\dot{\mathbf{r}}_3$ M1 A1ft

$\mathbf{v} = 30\ \mathbf{I} - 54\mathbf{j}$ (m s^{-1}) A1 3

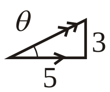
[8]

6. (a) I = 0.2[(15i + 15j) – (–10i)] M1

= 5i + 3j M1

$|I| = \sqrt{(5^2 + 3^2)} = \sqrt{34} = 5.8$ Ns M1 A1 4

(b)



$\tan\theta = \frac{3}{5} \Rightarrow \theta = 31°$ (nearest degree) M1 A1 ft 2

(c) KE Gain = $\frac{1}{2} \times 0.2[(15^2 + 15^2) - 10^2)] = 35$ J M1 A1 2

[8]

1. This question was well attempted by a majority of candidates.

In part (a) the most common incorrect answer was a sign error leading to an impulse of (5**i** + 12**j**) Ns rather than (5**i** – 12**j**). Some candidates failed to apply the impulse formula correctly, adding momentum rather than subtracting. Many students forgot to calculate the magnitude of their impulse. A few candidates started by finding the initial and final speeds of the ball and ignored the two dimensional nature of the problem, never producing a vector equation for the impulse or appropriate work using trigonometry.

In aprt (b) a common error was to find the angle for the initial velocity rather than the impulse. A minority of candidates were confused over which angle was required or made a trigonometric error, using the ratio $\frac{5}{12}$ rather than $\frac{12}{5}$ to find the angle.

For part (c) although there were many completely correct solutions, some candidates were unable to cope with using vectors to find speed and hence Kinetic Energy. The most common errors were, for example, to find $\sqrt{10^2 + 24^2}$ and then forget to square it or to attempt to square the vector velocity, treating $(10\mathbf{i} + 24\mathbf{j})^2$ as an algebraic expression and retaining **i** and **j** components in the answer.

2. Many candidates did not recognise this as a question on the impulse-momentum principle in vector form. Many of the weaker candidates simply worked with the given magnitudes. Some realised the need to resolve, but resolved and used only the component in the initial direction. Those candidates who resolved correctly had no problems with finding the magnitude of the impulse, though some left their answer as a vector.
Most candidates with an impulse (or change in velocity) in component form went on to find an angle. Unfortunately the majority of them found the supplementary angle, the angle to *BA* instead of *AB*, often without reference to a diagram with a marked angle.
Some candidates who struggled to find the impulse made a fresh start to find the angle, often drawing a correct vector triangle and using trigonometry to find the correct angle (or its supplement) without realising that the same diagram could have helped them with the impulse.

3. Candidates found this very accessible with the majority obtaining the correct velocity. Unfortunately many did not proceed to find the speed, which was a careless loss of two marks. Common errors included sign errors in the original equation, or in rearranging the equation, and errors in manipulating the fractions. Some candidates made the mistake of trying to work with the magnitudes of impulse and momentum.

4. Candidates made a confident start to this question, but in parts (a) and (b), poor algebraic skills and the lack of a clear diagram with the directions marked on it hampered weaker candidates' attempts to set up correct and consistent (or even physically possible) equations. The direction of *P* after impact was not given and those candidates who took its direction as reversed ran into problems when finding the value of *e*. Many realised that they had chosen the wrong direction and went on to answer part (b) correctly but some did not give an adequate explanation for a change of sign for their velocity of *P*. Algebraic and sign errors were common, and not helped by candidates' determination to reach the given answers.

Parts (c) and (d) caused the most problems. They could be answered using a wide variety of methods, some more formal than others. Many good solutions were seen but unclear reasoning and methods marred several attempts. Too many solutions were sloppy, with *u* or *d* appearing and disappearing through the working. A few words describing what was being calculated or expressed at each stage would have helped the clarity of solutions greatly. Students need to be reminded yet again that all necessary steps need to be shown when reaching a given answer. Too many simply stated the answer $\frac{3d}{5}$ without the explanation to support it.

5. Most candidates realised that they needed to differentiate, although there was the odd integration. Having found a velocity vector some then failed to find the modulus to obtain the speed. In part (b), a few used $\mathbf{I} = m(\mathbf{u} - \mathbf{v})$ and a very small number worked with scalars, but generally candidates reached the correct vector solution. Some candidates thought they then had to find the magnitude of their vector – they were not penalised for this.

6. (a) Many found the difference in the magnitudes of the momentum vectors instead of the other way round. Of those who knew the method for finding the impulse, many then forgot to find the magnitude of it.

(b) There was a good level of success with finding the angle although some forgot to round it to the nearest degree. The most common error was to get the tan upside down.

(c) The vectors here caused some confusion for the weaker candidates, with final answers often appearing as vectors.