AS/A Level Mathematics

Differential Equations

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.
• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
• **Fill in the boxes** at the top of this page with your name.
• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled..
• Answer the questions in the spaces provided
– there may be more space than you need.
• You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
• Answers should be given to three significant figures unless otherwise stated.

Information

• The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Try to answer every question.
• Check your answers if you have time at the end.

1 The annual rate of increase of a population is equal to 2% of the size of the population. y is the population in millions and t is the time in years.

(a) Write down a differential equation for this relationship **(2)**

(b) Show that $y = Ae^{0.02t}$ where A is a constant **(4)**

(c) Given that the initial population is 2.5 million. Find the population after 10 years. **(2)**

(Total for question 1 is 8 marks)

2 The rate of change of the temperature of a kettle of water (y) after it boils is directly proportional to the difference between the temperature of the water and the room temperature (20°C).

(a) Write down a differential equation for this relationship **(3)**

(b) Show that $y = 20 + Ae^{kt}$ where A and k are constants **(4)**

(c) Given that the initial temperature is 100°C write down the value of A. **(1)**

(d) After 8 minutes the temperature is 60°C show that $k = -\dfrac{1}{8}\ln 2$ **(3)**

(Total for question 2 is 11 marks)

3 (a) Express $\dfrac{3x - 3}{(x + 1)(2x - 1)}$ in partial fractions **(3)**

(b) Given that $x > 1$, find the general solution to the differential equation **(5)**

$$(x + 1)(2x - 1)\frac{dy}{dx} = y(3x - 3)$$

(c) Hence find the particular solution to the differential equation that satisfies $y = 6$ at $x = 5$, giving your answer in the form $y = f(x)$ **(4)**

(Total for question 3 is 12 marks)

4 Find the general solution to the differential equation $\dfrac{dy}{dx} = xy\sin x$

(Total for question 4 is 6 marks)

5 Find the general solution to the differential equation $\dfrac{dy}{dx} = y^2 \ln x$

(Total for question 5 is 6 marks)

6 Find the solution to the differential equation $\dfrac{dy}{dx} = (y + 1)^2$ given that when $y = 0$, $x = 2$

Give your answer in the form y = f(x)

(Total for question 6 is 6 marks)