Worksheet 1 Solutions

Question 1 Solution.

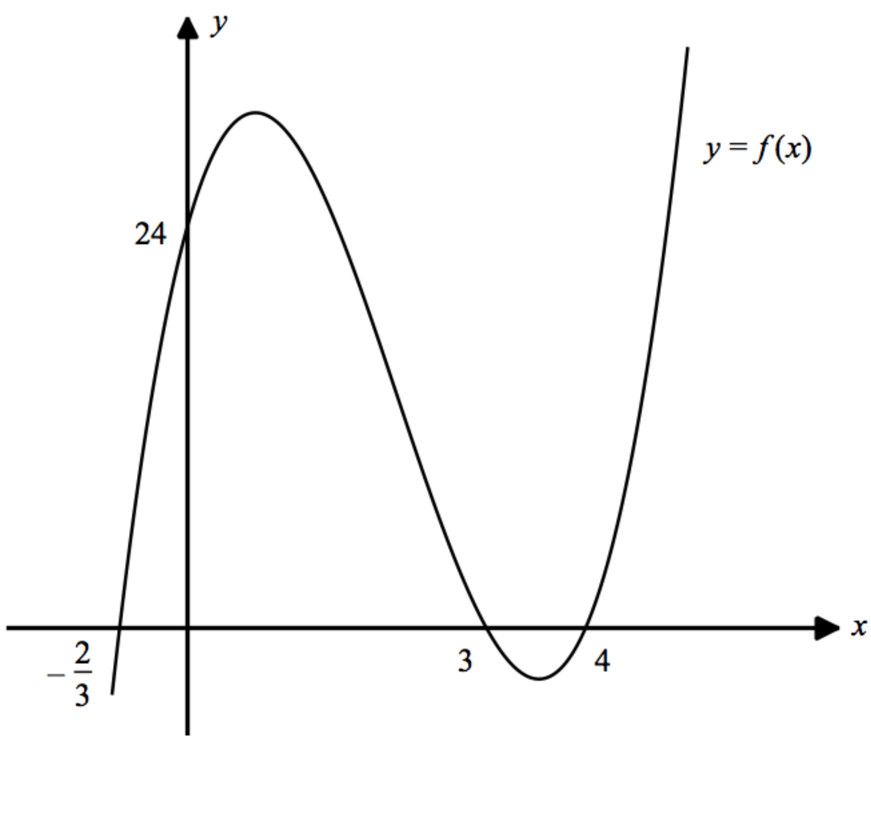
(a) Here we use the factor theorem:

$$\begin{aligned} f(3) &= 3(3)^3 - 19(3)^2 + 22(3) + 24 \\ &= 81 - 171 + 66 + 24 \\ &= -90 + 90 \\ &= 0 \end{aligned}$$

$\therefore$ By the factor theorem, $(x-3)$ is a factor of $f(x)$. [Alternatively, you could have used long division.]

(b) Using long division, we can show that $(3x+2)$ and $(x-4)$ are also factors of $f(x)$. Therefore the roots of the equation $f(x) = 0$ are $\boxed{x = -\frac{2}{3},\ x = 3 \text{ and } x = 4}$.

(c) Here is a sketch of the curve $y = f(x)$.



Question 2 Solution.

(a) To find the equation of a line, we need its gradient and a point it passes through. We have two points l_1 passes through, so what we need to compute is its gradient:

$$m_{l_1} = \frac{10-6}{-4-3} = -\frac{4}{7}$$

So the equation of l_1 is given by

$$y - 6 = -\frac{4}{7}(x-3)$$

$$\Rightarrow y = -\frac{4}{7}x + \frac{54}{7}$$

(b) (i) The gradient of l_2 is $\frac{7}{4}$. Since it passes through the point $(1, k)$, its equation can be given by

$$y - k = \frac{7}{4}(x-1) \qquad (1)$$

(1) is a sufficient form for the answer, since you haven't been asked to give it in a specific form, but we can also write it as

$$7x - 4y + (4k-7) = 0$$

(ii) When l_2 crosses the x-axis, $y = 0$, so

$$7x + 4k - 7 = 0 \Rightarrow x = \frac{7-4k}{7}$$

(iii) Similarly, when l_2 crosses the y-axis, $x = 0$, so

$$-4y + 4k - 7 = 0 \Rightarrow y = \frac{4k-7}{4}$$

(c) First, we have to be careful. One of the coordinates will be negative (since the gradient of our line is positive), so if we want to solve this by just multiplying the coordinates, we'll need to have an extra minus sign in the equation to account for this. Since the area of OAB is $\frac{1}{14}$, we can write

$$-\frac{1}{14} = \frac{1}{2}\left(\frac{7-4k}{7}\right)\left(\frac{4k-7}{4}\right)$$
$$-\frac{1}{7} = \frac{1}{28}(-16k^2 + 56k - 49)$$

$$16k^2 - 56k + 45 = 0$$

and if you solve this, you will find that $\boxed{k = \frac{5}{4}}$ or $\boxed{k = \frac{9}{4}}$.

Question 3 Solution.

(a) Firstly, the volume of the box is given by

$$V = x^2(2y+1)$$

Now we need to eliminate y from the expression, and to do this, we use the constraint given on the surface area of the box. We know that the surface area of the rectangular box is $2x^2 + 4x(2y+1)$ and so

$$2x^2 + 4x(2y+1) = 130$$
$$\Rightarrow x^2 + 2x(2y+1) = 65$$
$$\Rightarrow 2y + 1 = \frac{65}{2x} - \frac{1}{2}x$$

So the volume of the box can be given by

$$V = x^2\left(\frac{65}{2x} - \frac{1}{2}x\right) = \frac{65}{2}x - \frac{1}{2}x^3$$

(b) Differentiating V with respect to x gives

$$\frac{dV}{dx} = \frac{65}{2} - \frac{3}{2}x^2$$

Turning points occur when $\frac{dV}{dx} = 0$, i.e. when

$$x^2 = \frac{65}{3} \Rightarrow x = \pm\sqrt{\frac{65}{3}}$$

But $x > 0$, so the maximum occurs when $x = \sqrt{\frac{65}{3}}$. Thus, the maximum value of V is

$$V_{\max} = V\big|_{x=\sqrt{\frac{65}{3}}}$$

$$= \frac{65}{2}\sqrt{\frac{65}{3}} - \frac{1}{2}\left(\sqrt{\frac{65}{3}}\right)^3$$
$$= 100.852\ldots$$

so the maximum volume of the box is about $\boxed{101 \text{ cm}^3}$. [NB: this is clearly a maximum since V is a negative cubic.]

Question 4 Solution.

(a) $v = \sqrt{2(9.8)(20} = 19.798...$, so the ball hits the ground at $\boxed{20\text{ m/s}}$.

(b) Rebound speed $= 0.3 \times 19.798... = 5.939... \approx 5.9$ m/s, as required.

(c) First, the time taken for the ball to hit the ground for the first time is given by

$$20 = \frac{1}{2}(9.8)t^2 \Rightarrow t = \sqrt{\frac{40}{9.8}} = 2.020...$$

Now, we need to work out the time for the ball to reach the ground a second time after bouncing off the ground the first time. We can take 5.939... as the initial speed and, since we are interested in when the ball hits the ground again, the displacement will be 0.

$$0 = (5.939...)t + \frac{1}{2}(-9.8)t^2$$

$$\Rightarrow t(5.939... - 4.9t) = 0$$

and since $t \neq 0$, $t = 1.212...$s.

Therefore the total time for the ball to hit the ground a second time is $1.212... + 2.020... = 3.232...$, which is $\boxed{3.2\text{ seconds}}$ to two significant figures.

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