

$$1) \quad y = 4x$$

$$(x, 4x) \quad (x+h, 4(x+h))$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4(x+h) - 4x}{x+h - x} \\ &= \frac{4x + 4h - 4x}{h} \\ &= \frac{4h}{h} \\ &\underset{\cancel{h}}{=} 4 \end{aligned}$$

$$2) \quad y = x^3$$

$$(x, x^3) \quad (x+h, (x+h)^3)$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(x+h)^3 - x^3}{x+h - x} \\ &= \frac{(x+h)(x+h)(x+h) - x^3}{h} \\ &= \frac{(x+h)(x^2 + hx + h^2) - x^3}{h} \\ &= \frac{(x+h)(x^2 + 2hx + h^2) - x^3}{h} \\ &= \frac{x^3 + hx^2 + 2hx^2 + 2h^2x + h^2x + h^3 - x^3}{h} \\ &= \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h} \\ &= \underline{\underline{3x^2 + 3hx + h^2}} \end{aligned}$$

$$\lim_{h \rightarrow 0} m = \underline{\underline{3x^2}}$$

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$$y = 2x^3$$

$$(x, 2x^3) \quad (x+h, 2(x+h)^3)$$

$x_1 \quad y_1 \qquad \qquad x_2 \qquad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2(x+h)^3 - 2x^3}{x+h - x}$$

$$= \frac{2(x+h)(x+h)(x+h) - 2x^3}{h}$$

$$= \frac{2(x+h)(x^2 + 2hx^2 + h^2) - 2x^3}{h}$$

$$= \frac{2(x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3) - 2x^3}{h}$$

$$= \frac{2(x^3 + 3hx^2 + 3h^2x + h^3) - 2x^3}{h}$$

$$= \frac{2x^3 + 6hx^2 + 6h^2x + 2h^3 - 2x^3}{h}$$

$$= \frac{6hx^2 + 6h^2x + 2h^3}{h}$$

$$= 6x^2 + 6hx + 2h^2$$

$$\lim_{h \rightarrow 0} m = \underline{\underline{6x^2}}$$

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$$y = 5x^2$$

$$(x, 5x^2) \quad (x+h, 5(x+h)^2)$$

$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5(x+h)^2 - 5x^2}{x+h - x}$$

$$= \frac{5(x+h)(x+h) - 5x^2}{h}$$

$$= \frac{5(x^2 + hx + hx + h^2) - 5x^2}{h}$$

$$= \frac{5(x^2 + 2hx + h^2) - 5x^2}{h}$$

$$= \frac{5x^2 + 10hx + 5h^2 - 5x^2}{h}$$

$$= \frac{10hx + 5h^2}{h}$$

$$= 10x + 5h$$

$$\lim_{h \rightarrow 0} = 10x$$

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$$y = kx^3$$

$$\begin{array}{ll} (x, kx^3) & (x+h, k(x+h)^3) \\ x_1 & y_1 \\ & x_2 \\ & y_2 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{k(x+h)^3 - kx^3}{x+h - x}$$

$$= \frac{k(x+h)(x+h)(x+h) - kx^3}{h}$$

$$= \frac{k(x+h)(x^2 + hx + hx + h^2) - kx^3}{h}$$

$$= \frac{k(x+h)(x^2 + 2hx + h^2) - kx^3}{h}$$

$$= \frac{k(x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3) - kx^3}{h}$$

$$= \frac{k(x^3 + 3hx^2 + 3h^2x + h^3) - kx^3}{h}$$

$$= \frac{kx^3 + 3hkx^2 + 3h^2kx + h^3k - kx^3}{h}$$

$$= \frac{3hkx^2 + 3h^2kx + h^3k}{h}$$

$$= 3kx^2 + 3hkx + h^2k$$

$$\lim_{h \rightarrow 0} = \underline{\underline{3kx^2}}$$