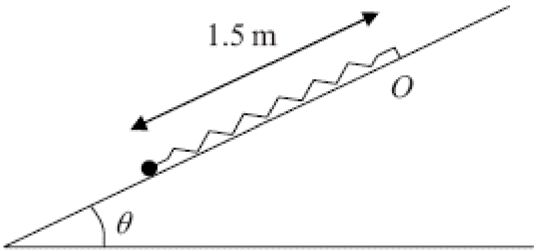
1.



A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in the diagram above. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(Total 9 marks)

2. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(a) Show that $v^2 = 2g(a+x) - \frac{3gx^2}{2a}$.

(4)

(b) Find the greatest speed attained by P as it falls.

(4)

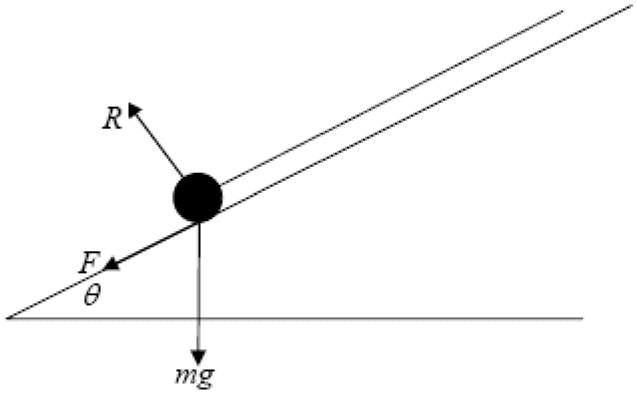
After release, P next comes to instantaneous rest at a point D.

(c) Find the magnitude of the acceleration of P at D.

(6)

(Total 14 marks)

1.



EPE lost $= \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} \left(= \frac{7}{36}\lambda \right)$ M1 A1

R$(\uparrow)$ $R = mg\cos\theta$ M1

$= 0.5g \times \frac{4}{5} = 0.4g$

$F = \mu R = 0.15 \times 0.4g$ M1 A1

P.E. gained = E.P.E. lost − work done against friction

$0.5\text{g} \times 0.7\sin\theta = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} - 0.15 \times 0.4g \times 0.7$ M1 A1 A1

$0.1944\lambda = 0.5 \times 9.8 \times 0.7 \times \frac{3}{5} + 0.15 \times 0.4 \times 9.8 \times 0.7$

$\lambda = 12.70......$

$\lambda = 13$ N or 12.7 A1

[9]

2. (a) $\frac{1}{2}mv^2 + \frac{3mgx^2}{4a} = mg(a + x)$ M1 A2 (1, 0)

leading to $v^2 = 2g(a + x) - \frac{3gx^2}{2a}$ * cso A1 4

(b) Greatest speed is when the acceleration is zero

$$T = \frac{\lambda x}{a} = \frac{3mgx}{2a} = mg \Rightarrow x = \frac{2a}{3}$$ M1 A1

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$ M1

$$v = \frac{2}{3}\sqrt{(6ag)}$$ accept exact equivalents A1 4

<u>Alternative</u>

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}$$

Differentiating with respect to x

$$2v\frac{dv}{dx} = 2g - \frac{3gx}{a}$$

$$\frac{dv}{dx} = 0 \Rightarrow x = \frac{2a}{3}$$ M1 A1

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$ M1

$$v = \frac{2}{3}\sqrt{(6ag)}$$ accept exact equivalents A1

(c) $v = 0 \Rightarrow 2g(a + x) - \dfrac{3gx^2}{2a} = 0$ M1

$3x^2 - 4ax - 4a^2 = (x - 2a)(3x + 2a) = 0$

$x = 2a$ M1 A1

At D, $m\ddot{x} = mg - \dfrac{\lambda \times 2a}{a}$ ft their $2a$ M1 A1ft

$|\ddot{x}| = 2g$ A1 6

<u>Alternative approach using SHM for (b) and (c)</u>

If SHM is used mark (b) and (c) together placing the marks in the gird as shown.

Establishment of equilibrium position

$$T = \frac{\lambda x}{a} = \frac{3mge}{2a} = mg \Rightarrow e = \frac{2a}{3}$$ bM1 bA1

N2L, using y for displacement from equilibrium position

$$m\ddot{y} = mg - \frac{\frac{3}{2}mg(y+e)}{a} = -\frac{3g}{2a}y$$ bM1 bA1

$$\omega^2 = \frac{3g}{2a}$$

Speed at end of free fall $u^2 = 2ga$ cM1

Using A for amplitude and $v^2 = \omega^2 (a^2 - x^2)$

$u^2 = 2ga$ when $y = -\frac{2}{3}a \Rightarrow 2ga = \frac{3g}{2a}\left(A^2 - \frac{4a^2}{9}\right)$ cM1

$$A = \frac{4a}{3}$$ cA1

Maximum speed $A\omega = \frac{4a}{3} \times \sqrt{\left(\frac{3g}{2a}\right)} = \frac{2}{3}\sqrt{(6ag)}$ cM1 cA1

Maximum acceleration $A\omega^2 = \frac{4a}{3} \times \frac{3g}{2a} = 2g$ cA1

[14]

1. This was probably the least well done of all the questions and correct solutions were relatively rare. The most common mistake was the assumption that there was no final EPE but this was often combined with other errors to give a huge variety of different wrong answers. A surprisingly large proportion of candidates treated this as an equilibrium question, either starting with $T = \mu R + mg \sin \theta$ or slipping an EPE term in as well for good measure. Others realised that it was an energy question but forgot to include the work done against friction; these attempts either used only the frictional force in their equation or ignored it completely, offering as their solution "Initial EPE = *mgh*". Another common error was to include the GPE term twice, once as energy and again as part of the "Work done" expression, showing a lack of understanding of the origin of the *mgh* formula. Very many candidates scored only the 3 marks for finding friction, while those who thought that this was a simple conversion of EPE into GPE had no need to find the friction and so didn't even earn these. Some candidates who included all necessary terms fell at the accuracy hurdle. Inexplicably, a final extension/ compression of 0.2 was not uncommon and other errors arose from inappropriate use of the various lengths mentioned, 1.5, 0.9 and 0.7. There were also all the usual sign errors generated by mistakes in identifying gains and losses. A few candidates produced a perfect solution but lost the final mark by giving their answer as 12.7008.

2. Most candidates managed to arrive at the required result in part (a), though some unnecessarily split the motion into two parts, considering freefall initially to find the kinetic energy when the string became taut and then proceeding to consider the taut string and others would clearly have failed had not the answer been provided.

Parts (b) and (c) were often difficult to disentangle. Some candidates took an SHM approach from the start of to the Examiner". The main fault was not when to start considering SHM but not establishing a correct equation to prove that the motion was SHM; no credit is given for making assumptions of this nature. A fully correct solution using SHM was rare, the equations frequently being unsatisfactory due to using x for the distance from the equilibrium point and confusing it with x as defined in the question to be the extension of the string.

For the non-SHM solutions, in part (b) many candidates assumed that the maximum speed occurred when $x = 0$ rather than when $a = 0$. In part (c) most substituted $v = 0$ in the result from part (a). Some did not expect to obtain a quadratic and so stopped working (or ran out of time?). Of those who obtained a solution for their quadratic equation, some would then try incorrectly to use their value for x as the amplitude in SHM instead of using an equation of motion and Hooke's law. Many equations of motion omitted the weight of the particle.