38 a $x = 0.421, 2.72, 3.56, 5.86$

b $x = 1.85$

39 a $\text{LHS} \equiv 3(1+\tan^2\theta) - 7\tan^2\theta$

$\equiv 3 + 3\tan^2\theta - 7\tan^2\theta$

$\equiv 3 - 4\tan^2\theta$

$\equiv \text{RHS}$

b $\theta = 0.669, 3.81, 2.47$

40 a $\cos 2x \equiv \cos x\cos x - \sin x\sin x$

$\equiv \cos^2 x - \sin^2 x$

$\equiv 1 - 2\sin^2 x$ as required

b $x = 90°, -30°, -150°$

41 a $\sqrt{17}\sin(\theta + 0.245)$

b Max is $\sqrt{17}$

Occurs when $\theta = 1.33°$

42 $-3 < x < 1$

43 $y = 2x + 1$

44 a $\dfrac{(x+2)\cos x - \sin x}{(x+2)^2}$

b $\dfrac{(x+2)\cos x - \sin x}{(x+2)^2} = 0$

$\Rightarrow (x+2)\cos x - \sin x = 0$

$\Rightarrow (x+2) = \tan x$

$\Rightarrow \arctan(x+2) = x$ as required

c $x = 1.27$

45 a $\dfrac{1}{t}$

b $y - 5 = -2(x-1)$

c $x = \left(\dfrac{y-1}{2}\right)^2 - 3$

46 When $y = 0$, $\dfrac{dy}{dx} = \dfrac{2}{3}$

When $y = \dfrac{3}{2}$, $\dfrac{dy}{dx} = \dfrac{5}{6}$

47 $\cos(x+h) - \cos x$

$= \cos x\cos h - \sin x\sin h - \cos x$

when h small

$\approx \cos x\left(1 - \dfrac{h^2}{2}\right) - h\sin x - \cos x$

$= -\dfrac{h^2}{2}\cos x - h\sin x$

$\dfrac{dy}{dx} = \lim\limits_{h\to 0}\left(\dfrac{\cos(x+h) - \cos x}{(x+h) - x}\right)$

$= \lim\limits_{h\to 0}\left(\dfrac{-\dfrac{h^2}{2}\cos x - h\sin x}{h}\right)$

$= \lim\limits_{h\to 0}\left(-\dfrac{h}{2}\cos x - \sin x\right)$

$= -\sin x$

48 a **i** $-x\cos x + \sin x\ (+c)$

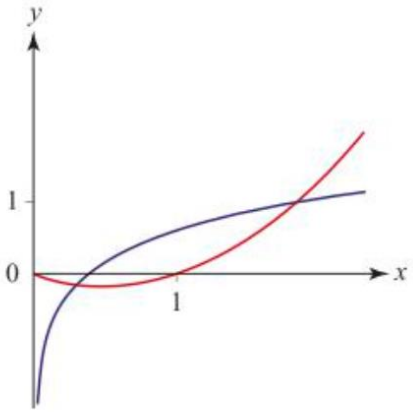
ii $x\ln x - x\ (+c)$

b $-\dfrac{1}{4}(x^2+1)^{-2}(+c)$

49 $\ln\left(\frac{27}{4}\right)$

50 $y = e^{\sin x - \frac{1}{2}}$

51 a



b $x = 0.298$ (or $x = 1.87$)

c e.g.
$0.2975^2 - 0.2975 - \ln 0.2975 - 1 = 0.003$
$0.2985^2 - 0.2985 - \ln 0.2985 - 1 = -0.0004$
Change of sign so root in interval, hence solution is correct to 3 significant figures.

52 a $x = -1.343$

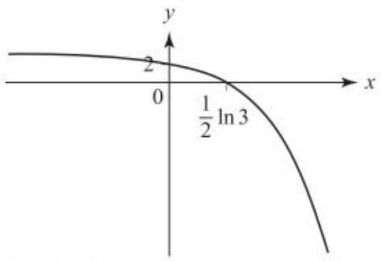
b 1.9 is near a stationary point so process could be unstable/next approximation is 19.6 which is nowhere near a root.

53 a One solution as the graphs intersect once

b Let $f(x) = \sqrt{x} - e^{-x}$
Then $f(0.4) = \sqrt{0.4} - e^{-0.4}$ $(= -0.04)$
$f(0.5) = \sqrt{0.5} - e^{-0.5} (= 0.1)$
Change of sign so solution is between 0.4 and 0.5

c $x = 0.43$
$f(0.425) = \sqrt{0.425} - e^{-0.425}$ $(= -0.002)$
$f(0.435) = \sqrt{0.435} - e^{-0.435}$ $(= 0.012)$
Change of sign so 0.43 is correct to 2 dp

54 a



Asymptote at $y = 3$

b $f^{-1}(x) = \frac{1}{2}\ln(3 - x)$

Domain is $x < 3$
Range is $f^{-1}(x) \in \mathbb{R}$

55 $\sqrt{27}$

56 $d = 2.5, a = 1$

57 $\frac{12}{5}$ or 2.4

58 a **i** Geometric, $a=\frac{2}{3}$, $r=\frac{1}{3}$

ii 1

b Arithmetic, $\frac{n}{2}(n+1)(\ln 3)$

59 a $2-\frac{5}{12}x-\frac{25}{288}x^2+\ldots$ **b** 19.9582

60 Let $\frac{A+B}{2}=P$ and $\frac{A-B}{2}=Q$

Then $P+Q=A$ and $P-Q=B$

$\text{LHS}=\cos(P+Q)-\cos(P-Q)$

$\equiv(\cos P\cos Q-\sin P\sin Q)-(\cos P\cos Q+\sin P\sin Q)$

$\equiv -2\sin P\sin Q$

So $K=-2$

61 $0.554\le x\le 2.12$

62 $x=214.7°, 325.3°$

63 $\left(53.1°, \frac{1}{5}\right)$

64 a LHS

$\equiv(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$\equiv 1-2(\sin x\cos x)^2$

$\equiv 1-2\left(\frac{1}{2}\sin 2x\right)^2$

$\equiv 1-2\left(\frac{1}{4}\sin^2 2x\right)$

$\equiv 1-\frac{1}{2}\sin^2 2x$

$\equiv 1-\frac{1}{2}\left(\frac{1-\cos 4x}{2}\right)$

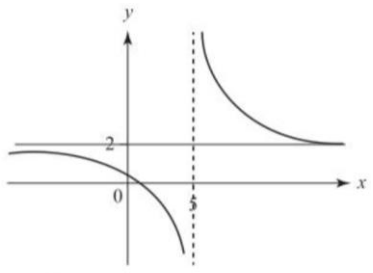
$\equiv 1-\frac{1}{4}(1-\cos 4x)$

$\equiv \frac{1}{4}(4-1+\cos 4x)$

$\equiv \frac{1}{4}(3+\cos 4x)\equiv \text{RHS}$

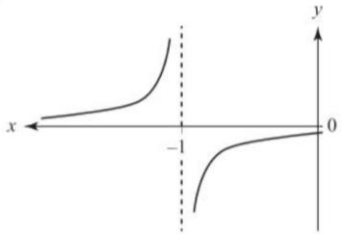
b $x=\pm 0.48, \pm 1.09, \pm 2.05, \pm 2.66$

65 a **i**



$x=5, y=2$

ii



$x=-1, y=0$

b $b=-4, a=8$

c **i** $y=\frac{2x-2}{x-5}$ **ii** $y=-\frac{8}{1+x}$

66 **a** $\frac{2}{27}$ **b** $y=\frac{1}{2}\sqrt{x}+10+\frac{50}{\sqrt{x}}$

67 $\frac{5}{2}, -\frac{20}{23}$

68 149 cm^2, $r = 9.73$ cm

69 $-\ln 2$

70 Let $y = \arccos 3x$

Then $\cos y = 3x$

$-\sin y \frac{dy}{dx} = 3$

$\Rightarrow \frac{dy}{dx} = -\frac{3}{\sin y}$

$= -\frac{3}{\sqrt{1-\cos^2 y}}$

$= -\frac{3}{\sqrt{1-(3x)^2}}$

$= -\frac{3}{\sqrt{1-9x^2}}$ as required

71 $f(\theta) = \int \frac{\sin\theta}{\cos\theta}\, d\theta$

$= -\ln|\cos\theta| + c$

$= \ln|\cos\theta|^{-1} + c$

$= \ln|\sec\theta| + c$

$f\left(\frac{\pi}{3}\right) = \ln 6$

$\Rightarrow \ln 2 + c = \ln 6$

$\Rightarrow c = \ln 3$

$\therefore f(\theta) = \ln|\sec\theta| + \ln 3$

72 **a** $ye^y - e^y = \frac{1}{2}e^{2x} + c$

b $x = \frac{1}{2}\ln(2ye^y - 2e^y + 1)$

73 $\frac{1}{216}(3x-1)^8 + \frac{2}{189}(3x-1)^7 + \frac{1}{162}(3x-1)^6 (+c)$

74 4

75 **a** 3.955

b Not true, estimation is an over-estimate since curve is convex.

76 13.5 square units

77 $\frac{\pi}{4} + \frac{\sqrt{3}}{4}$