YEAR 2 PURE — INTEGRATION — Chapter 11

BASICS (must Learn)

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (ax+b)^n = \frac{1}{a}\frac{1}{n+1}(ax+b)^{n+1} + C$$

$$\int e^{ax+b} = \frac{1}{a}e^{ax+b} + C$$

RECIPROCALS (2 types)

$$\int \frac{1}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$

OR

$$\int \frac{1}{(ax+b)^n} = \int (ax+b)^{-n} = \frac{1}{a}\frac{(ax+b)^{-n+1}}{-n+1} + C$$

TRIG FUNCTIONS

$$\int \cos(ax+b) = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sin(ax+b) = -\frac{1}{a}\cos(ax+b) + C$$

All others given in the formula booklet

use Differentiation list and Integration list.

Dont forget to multiply by $\frac{1}{k}$ and $+C$

REVERSE CHAIN RULE

- Two functions multiplying together

if $\int f'(x)\ f(x)^n$ possible answer $f(x)^{n+1}$ then check to see if you need a multipler

- Two Functions dividing

if $\int \frac{f'(x)}{f(x)}$ possible answer $\ln|f(x)|$ check for multipler

BY SUBSTITUTION (Replace x terms wit u terms)

STEP 1 : work out x and dx
STEP 2 : Substite these into orignal expressions
STEP 3 : Integrate simplified expressions
STEP 4 : Write answer back in terms of x

BY PARTS (TWO FUNCTIONS MULTIPLYING)

$$\int u \frac{dv}{dx}\,dx = uv - \int v \frac{du}{dx}\,dx$$

Let $u = x^n$ term UNLESS one term is $\ln|x|$

Special Case : $\int \ln x = \int 1 \times \ln x$ where $u = \ln x$ and $\frac{dv}{dx} = 1$

$$\int \ln x = x \ln x - x + c$$

TRIG FUNCTIONS (IN THE FORMULA BOOKLET)

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec kx$	$k\sec kx \tan kx$
$\cot kx$	$-k\operatorname{cosec}^2 kx$
$\operatorname{cosec} kx$	$-k\operatorname{cosec} kx \cot kx$

Integration (+ constant)

$f(x)$	$\int f(x)\,dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Dont forget to multiply by $\frac{1}{k}$ and $+C$

example:

$$\int \sec^2 3x = \frac{1}{3}\tan 3x + C$$

PARTIAL FRACTIONS

$$\int \frac{A}{(x+1)} + \frac{B}{(x-1)} = A\ln|x+1| + B\ln|x-1| + C$$

$$\int \frac{A}{(3x+1)} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)} = \frac{1}{3}A\ln|3x+1| + B\frac{(x-2)^{-1}}{-1} + C\ln|x-2| + C$$

ALGEBRAIC FRACTIONS

watch out for IMPROPER FRACTIONS (numerator power $\geq$ denominator power)

$$\int \frac{9x^2-3x+2}{9x^2-4} = \int 1 + \frac{6-3x}{9x^2-4} = \int 1 + \frac{1}{3x-2} - \frac{2}{3x+2}$$

long division

partial fractions

DIFFERENTIAL EQUATION

mix of variables and derivates $\frac{dy}{dx} = xy$

STEP1 : Get x and y variables and derivates on separate sides

$$\int \frac{1}{y}\,dy = \int x\,dx$$

STEP2: Integrate both sides

$$\ln y = \frac{x^2}{2} + c$$

STEP3: Make y the subject

$$y = e^{\frac{1}{2}x^2 + c}$$

STEP 4: Simplify if possible

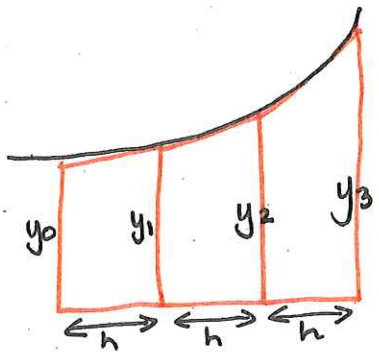
$$y = e^{\frac{1}{2}x^2} e^{c} \qquad \text{let } e^c = A$$

$$y = Ae^{\frac{1}{2}x^2}$$

TRAPEZIUM RULE (Estimate Area Under the graph)

$$\int_b^a y\,dx \approx \frac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

1st height — Last height — all height inbetween



PARAMETRIC EQUATIONS (Area under the graph)

Cartesian Equation

$$\text{AREA} = \int_{x\,min}^{x\,max} y\,dx$$

Parametric Equation

$$\text{AREA} = \int_{t\,min}^{t\,max} y\,\frac{dx}{dt}\,dt$$

Dont forget to change the limits

WHICH INTEGRATION CHECK LIST

- NOT TRIG

  single function
  - Basic
  - Reciprocal

  multiplying functions
  - Reverse Chain
  - Substitution
  - by parts

  dividing functions
  - Reverse Chain
  - Substitution
  - Partial Fractions / Algebraic Fraction (improper)

- TRIG

  single function
  - Basic
  - Formula Booklet
  - Rewrite using trig indentities

  multiplying function
  - Formula Booklet
  - Reverse chain rule
  - by substitution
  - by parts
  - Rewrite using trig indentities

  dividing trig functions
  - Reverse Chain Rule
  - by substitution
  - Rewrite using trig identities