CM

A Level Maths Question Countdown

9 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self

Pure questions – standard

1 (a) Find the **set** of values of x such that the curve with equation $y = \dfrac{2x}{x^2+3}$ is increasing.

(b) Given that $y = \dfrac{e^{-x^2}}{1+x^2}$, show that

$$\frac{dy}{dx} = -2xy(1+ye^{x^2})$$

NB: part (b) for Q1 is not standard. It is a challenging exercise.

2 (a) Given a natural number n and an irrational number s, prove that $\dfrac{n}{s}$ is irrational.

(b) Prove that there are infinitely many primes.

Pure questions – problems

3 The function f is defined such that

$$f: x \mapsto \frac{4x-1}{7x+2}, \quad x > -\frac{2}{7}$$

(a) Write down the range of f.

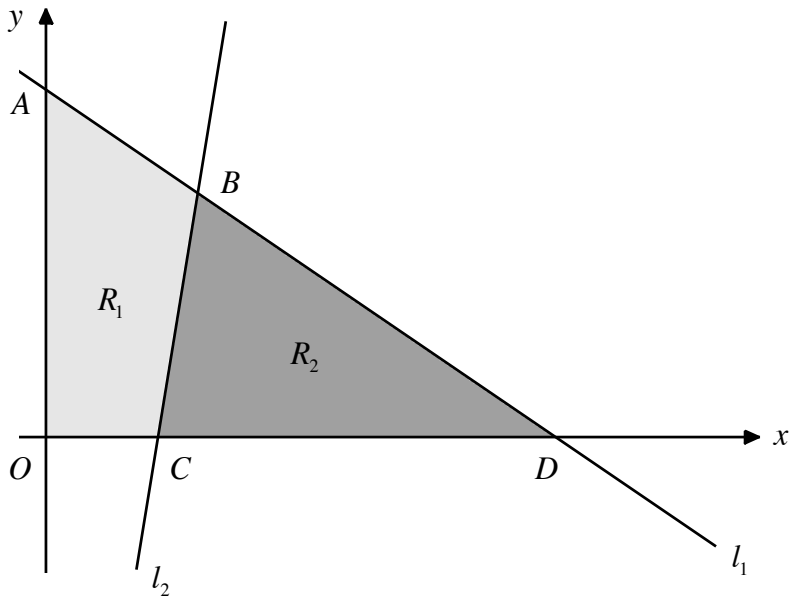
(b) Find $f^{-1}(x)$ **and** state its domain and range.

The function g is defined such that

$$g(x) = \ln(x+1), \quad x > -1$$

(c) Find, in terms of e, the exact solution to the equation $gf(x) = 5$.

4



The diagram above shows two straight lines l_1 and l_2. The line l_1 has the equation $x + 3y = 15$ and meets the y axis at the point A and the x axis at the point D.

The line l_2 is perpendicular to l_1 and passes through the point $(4, 2)$.

(a) Find the equation of l_2, giving your answer in the form $y = mx + c$.

The lines l_1 and l_2 intersect at the point B.

The region R_1 is bounded by the quadrilateral $OABC$ and the region R_2 is bounded by the triangle BCD.

(b) Showing your method clearly, determine the ratio

$$\text{area of } R_1 : \text{area of } R_2$$

giving your ratio in its simplest form.

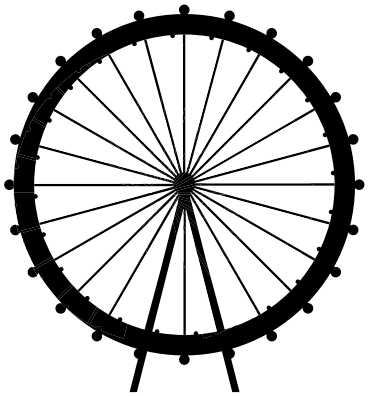
Pure questions – modelling

5 (a) Express $6\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give your value of R as an exact value and your value of α to two decimal places.

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning is modelled by the equation

$$H = 30 - 6\cos\left(\frac{\pi}{3}t\right) + 8\sin\left(\frac{\pi}{3}t\right), \quad t \geqslant 0$$



The diagram shows an illustration of the Ferris wheel.

(b) Using the model, find

(i) the initial height of the passenger,

(ii) the radius of the Ferris wheel.

(c) Determine the duration of one cycle of the Ferris wheel.
(Solutions based entirely on graphical or numerical methods are not acceptable.)

Applied questions – mechanics

6 A particle P is moving in a plane. At time t seconds, its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = (6\mathbf{t} - 4)\mathbf{i} + \left(-\frac{5}{2}t^2\right)\mathbf{j}, \quad t \geqslant 0$$

The initial displacement of P from the origin O is $3\mathbf{i} - 2\mathbf{j}$.

(a) Find the displacement of P from O at the time t.

(b) Hence find the distance of P from O at the instant when it is moving parallel to $\mathbf{j}$.
Give your answer to two decimal places.

Applied questions – statistics

7 A drug company claims that only 2% of individuals experience side effects from use of their drug.

To test their claim, 100 indivdiuals take the drug and the company monitors their reaction to the drug.

(a) Define a suitable distribution to model the number of individuals in the sample that experience side effects from the drug.

Using your answer to part (a),

(b) calculate the probability that more than 5 people in the sample experience side effects.

Tina believes the drug company is wrong and that more than 2% of individuals experience side effects from the use of the drug.

Tina takes a sample of 1000 users of the drug.

(c) Write down suitable hypotheses for Tina's hypothesis test.

She finds that 28 individuals experience side effects from the drug.

(d) Using a normal approximation, find the p-value for the hypothesis test.

(e) Hence comment on Tina's claim at the 1% level of significance.

(f) State **one** limitation of hypothesis testing.