1 A sequence is defined by the recurrence relation $u_{n+1} = 1 - \frac{1}{u_n}$, where $u_1 = 2$

a Write down the values of

i u_2

ii u_3

iii u_4 **[6 marks]**

b Deduce the value of u_{50} **[2]**

2 Write the first four terms of each sequence, then describe the sequences as either increasing, decreasing or periodic.

a $u_n = 2\cos(180n)°$ **[3]**

b $u_n = 0.2^n + 4$ **[3]**

c $u_n = n^2 + 4n - 2$ **[3]**

3 Write down the first four terms in the binomial expansion, in ascending powers of x, of $(1-2x)^{-2}$, stating the values of x for which the expansion is valid. **[6]**

4 A car costs £30 000. Its value depreciates by 20% per annum. Work out

a Its value after 1 year, **[1]**

b Its value after 4 years, **[2]**

c The year in which it will be worth less than £5000 **[4]**

5 A sequence of terms is defined by the recurrence relation $u_{n+1} = 4 - ku_n$, where k is a constant.

Given that $u_1 = 3$

a Work out an expression in terms of k for u_2 **[2]**

b Work out an expression in terms of k for u_3 **[2]**

Given also that $u_1 + u_2 + u_3 = 9$

c Calculate the possible values of k **[4]**

6 The sum to infinity of a geometric series is 20. The first term is 4

a Calculate the common ratio of the series. **[3]**

b Evaluate the third term of the series. **[2]**

7 Adam plans to pay money into a savings scheme each year for 20 years. He will pay £800 in the first year, and every year he will increase the amount that he pays into the scheme by £100

a Show that he will pay £1000 into the scheme in year 3 **[1]**

b Calculate the total amount of money that he will pay into the scheme over the 20 years. **[2]**

c Over the same 20 years, Ben will also pay money into a savings scheme. He will pay £610 in the first year, and every year he will increase the amount that he pays into the scheme by £d. Given that Adam and Ben will pay in exactly the same total amounts over the 20 years, calculate the value of d **[3]**

8 When $(1+ax)^n$ is expanded the coefficients of x and x^2 are -4 and 20 respectively.

a Work out the value of a and the value of n **[8]**

b Evaluate the coefficient of x^3 **[2]**

9 The second term of a geometric series is 120 and the fifth term is 15. Work out

a The common ratio of the series, **[4]**

b The first term of the series, **[1]**

c The sum to infinity of the series. **[2]**

10 a Use a formula to evaluate $\sum_{r=1}^{40}(3r+1)$ **[3]**

b Calculate the value of n for which $\sum_{r=1}^{n}(3r+1)=9800$ **[4]**

11 a Write down the first three terms in the binomial expansion of $(1-2x)^{\frac{1}{2}}$, in ascending powers of x **[3]**

b Write down the first three terms in the binomial expansion of $(1+x)^{-\frac{1}{2}}$, in ascending powers of x **[3]**

c Use your answers to **a** and **b** to prove that $\sqrt{\frac{1-2x}{1+x}}=1-\frac{3}{2}x+\frac{3}{8}x^2+\ldots$ **[4]**

12 The fourth term of an arithmetic series is 11 and the sum of the first three terms is -3

a Write down the first term of the series. **[4]**

b Work out the common difference of the series. **[1]**

c Given that the sum of the first n terms of the series is greater than 500, calculate the least possible value of n **[5]**

13 The first three terms of a geometric series are $(3p-1)$, $(p-3)$ and $(2p)$ respectively.

a Use algebra to work out the possible values of p **[5]**

b For the negative value of p, calculate the sum to infinity of the series. **[3]**

c For the positive value of p, evaluate the sum of the first 999 terms of the series. **[2]**

14 a Write down the first four terms in the binomial expansion $\sqrt{1-x}$, in ascending powers of x **[6]**

b By substituting $x=\frac{1}{4}$, work out a fraction that is an approximation to $\sqrt{3}$ **[4]**

15 A salesman sells vacuum cleaners for £120 each. In one week, he receives 2% commission on the first vacuum cleaner he sells, 4% commission on the second vacuum cleaner he sells, with commission increasing in steps of 2% , so that he receives commission of 30% on the sale of his fifteenth vacuum cleaner. Commission stays fixed at 30% for the sale of all vacuum cleaners, after the sale of his fifteenth vacuum cleaner in that week.

a Calculate how much commission he receives in a week for the sale of

i His first vacuum cleaner,

ii His fifth vacuum cleaner,

iii His twentieth vacuum cleaner. **[6]**

In one week he sells 40 vacuum cleaners.

b How much commission does he receive in total that week? **[5]**

16 The sum to infinity of a geometric series is 48, and the sum of the first two terms of the series is 45

The common ratio of the series is r

a Prove that r satisfies the equation $1-16r^2=0$ **[4]**

b Calculate the sum of the first four terms of the series. **[4]**

17 The training programme of a cyclist requires her to cycle 3 km on the first day of training.

Then, on each day that follows, she cycles 2 km more than she cycled on the day before.

a Calculate how far she cycles on the seventh day. **[1]**

b Calculate the total distance she has cycled by the end of the tenth day. **[2]**

c On which day of training will she cycle more than 100 km? **[3]**

d On which day of training will the total distance that she has cycled exceed 1000 km? **[5]**

18 a Write down the first three terms in the binomial expansion of $\sqrt{4-x}$, in ascending powers of x **[7]**

b Deduce an approximate value of $\sqrt{399}$, giving your answer to 3 decimal places. **[5]**

19 An investment scheme pays 3% compound interest per annum. The interest is paid annually.

A deposit of £1000 is invested in this scheme at the start of each year.

The initial investment of £1000 is made at the start of year 1

a Explain why the value of the investment at the start of year 2 is £2030 **[2]**

b Calculate the value of the investment at the start of year 3 **[2]**

c Work out the year in which the total value of the investment exceeds £50 000 **[4]**

20 The sum of the first two terms of an arithmetic series is 2. The sum of the first ten terms of the series is 330

a Work out the common difference of the series. **[5]**

b Write down the first term of the series. **[1]**

c Given that the sum of the first n terms of the series is equal to 1170, find the value of n **[4]**

21 Given that $f(x) = \frac{5x}{(2+x)(1-2x)} \equiv \frac{A}{2+x} + \frac{B}{1-2x}$

a Work out the values of the constants, A and B **[5]**

b Write down the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3 **[11]**

c State the values of x for which the expansion is valid. **[1]**

22 Given that $f(x) = \frac{13x-33}{(5-x)(1+3x)}$

a Work out the expansion of $f(x)$ up to and including the term in x^3 **[14]**

b State the values of x for which the expansion is valid. **[1]**

23 When a ball is dropped from a height of h metres above a hard floor it rebounds to a height of $\frac{3}{4}h$

A ball is dropped from an initial height of 2 metres. Calculate

a The height to which the ball rises after the first bounce, **[2]**

b The total distance the ball has travelled when it hits the floor for the second time, **[2]**

c The total distance that the ball travels. **[3]**

24 Given that x, 15 and y are consecutive terms of an arithmetic series, and 1, x and y are consecutive terms of a geometric series, work out the possible values of x and y **[9]**

25 By solving an equation, find the limit of these sequences as $n \to \infty$. Where appropriate, give answers in simplified surd form.

a $u_{n+1} = 0.2u_n + 4$ **[2]**

b $u_{n+1} = 9 - 0.2u_n$ **[2]**

c $u_{n+1} = \frac{1}{2}\left(\frac{1}{3}u_n - 10\right)$ **[2]**

d $u_{n+1} = \left(\sqrt{2} - 1\right)u_n + 4$ **[2]**

e $u_{n+1} = \frac{1}{\sqrt{2}}u_n + \sqrt{2}$ **[2]**

f $u_{n+1} = 0.5u_n^2 + 0.5$ **[2]**