Algebra and Functions 1

Algebra is a pretty important part of maths — so it's a good idea to get to grips with it now. Some of this will be stuff you came across at GCSE and some of it will be brand spanking new. Ooooh, exciting.

1 Prove that $x^5 \geq 5^x$ for the integers $2 \leq x \leq 5$.

(2 marks)

2 Find a counter example to disprove the following statement:

The sum of any two prime numbers is even.

(2 marks)

3 Prove that, when n is an odd integer, $3n^2 - 12$ is always odd, and when n is an even integer, $3n^2 - 12$ is always even.

(3 marks)

4 Find a counter example to disprove the following statement:

If $x > y$, then $\frac{x}{y} > \frac{y}{x}$.

(2 marks)

5 Prove that $n^3 + 2n^2 + 12n$ always has a factor of 8 when n is even.

(3 marks)

Algebra and Functions 1

6 Prove that $x(x + 2y + 4) \geq 4x - y^2$ for any integers x and y.

(2 marks)

7 Prove that the difference between the cube and square of any integer is always even.

(4 marks)

8 Prove that the product of any two distinct prime numbers has exactly four factors.

Think about the factors of each prime number first.

(3 marks)

9 Write down the exact value of $36^{-\frac{1}{2}}$.

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(1 mark)

10 Simplify $\frac{a^6 \times a^3}{\sqrt{a^4}} \div a^{\frac{1}{2}}$.

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(2 marks)

Algebra and Functions 2

17 $(x - 1)(x^2 + x + 1) = 2x^2 - 17$

a) Rewrite the equation above in the form $f(x) = 0$, where $f(x)$ is of the form $f(x) = ax^3 + bx^2 + cx + d$.

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(2 marks)

b) Show that $(x + 2)$ is a factor of $f(x)$.

(2 marks)

c) Hence write $f(x)$ as the product of a linear factor and a quadratic factor.

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(2 marks)

d) By completing the square, or otherwise, show that $f(x) = 0$ has only one root.

(2 marks)

18 A function is defined by $f(x) = x^3 - 4x^2 - ax + 10$. $(x - 1)$ is a factor of $f(x)$.
Find the value of a and hence or otherwise solve the equation $x^3 - 4x^2 - ax + 10 = 0$.

Use the factor theorem to find the value of a.

$a =$ ..... $x =$ .....

(6 marks)

EXAM TIP

Quadratics have a habit of popping up in exam questions where you least expect them (like in exponentials, trig equations or mechanics), not to mention in simultaneous equations, inequalities and cubics — so make sure you can handle them. It's worth practising your factorising skills, as it could save you a lot of time in the exam, and time is ~~money~~ marks.

Score