1 Integrate each of these expressions.

a $\frac{1}{x}$ **[1]** **b** $\sin(x-3)$ **[2]** **c** e^{2x} **[2]** **d** $\cos 2x$ **[2]**

2 $f'(x)=2+3\sin 6x$

Find the equation of the curve $y=f(x)$, which passes through the point (0, 1) **[4]**

3 **a** Find these integrals.

i $\int \frac{1}{x+3}\,dx$ **ii** $\int \sin x\cos^2 x\,dx$ **iii** $\int 2x(x^2+4)^3\,dx$ **[5]**

b Find the exact value of $\int_0^1 \frac{x}{x^2+1}\,dx$. Show your working. **[4]**

4 **a** Use integration by parts to calculate each of these integrals.

i $\int xe^x\,dx$

ii $\int x\sin x\,dx$ **[5]**

b Show that the integral $\int_0^{\frac{\pi}{6}} x\sin 2x\,dx$ can be written in the form $\beta\sqrt{3}-\alpha\pi$

where α and β are constants to be found. **[4]**

5 **a** Find the general solution to the differential equation $\frac{dy}{dx}=\frac{y}{x+1}$

Give your answer in the form $y=f(x)$ **[4]**

b Find the particular solution for curve $y=f(x)$ which passes through the point (1, 8) **[2]**

6 **a** Express $\frac{10-13x}{(3+x)(1-2x)}$ in the form $\frac{A}{3+x}+\frac{B}{1-2x}$, where A and B are integers. **[3]**

b Hence find the area bounded by the curve $y=\frac{10-13x}{(3+x)(1-2x)}$, the x-axis and the lines $x=-1$ and $x=0$. Show your working. **[4]**

7 **a** Use the substitution $u=1+2x$ to find $\int \frac{2x}{1+2x}\,dx$. Give your answer in terms of x **[5]**

b Find the exact area of the region bounded by the curve $y=\frac{2x}{1+2x}$, the x-axis and the lines $x=0$ and $x=1$. Show your working. **[2]**

8 Find the value of $\int_1^2 \frac{4x+3}{2x^2+3x-2}\,dx$. Show your working. **[6]**

9 $f(x)=\frac{6x^3+14x^2+11x-1}{3x^2+7x+2}$

a Given that $f(x)$ can be expressed as $Ax+\frac{B}{x+2}+\frac{C}{3x+1}$, find the values of A, B and C **[4]**

The gradient of a curve is given by $\frac{dy}{dx}=f(x)$

b Find the equation of the curve given that it passes through the point (0, 0) **[4]**

10 Find the general solution to the differential equation $\frac{dy}{dx}=\frac{5y}{2-3x-2x^2}$

Give your answer in the form $y=f(x)$ **[7]**

11 A radioactive material decays such that the rate of change of the number, N, of particles is proportional to the number of particles at time t days.

a Write down a differential equation in N and t **[1]**

Initially there are N_0 particles and it takes T days for N to halve.

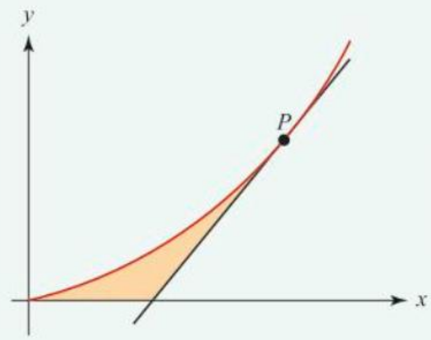
b Solve your differential equation, giving the solution in terms of N_0, t and T, with N as the subject. **[6]**

12 a Sketch the graphs of $y = \sin 2x$ and $y = \cos x$ on the same axes for $0 \le x \le \pi$
Give the points of intersection with the coordinate axes. **[4]**

b Find the values of x in the interval $[0, \pi]$ of the points of intersection of the two curves. Show your working. **[4]**

c Calculate the total area enclosed between the two curves in the interval $[0, \pi]$. Show your working. **[4]**

13 The curve C has a tangent at the point P as shown.



The equation of C is $y = x\mathrm{e}^{\frac{x}{3}}$ and P has x-coordinate 3

a Find the equation of the tangent to the curve at P. Show all your working. **[4]**

b Work out $\int x\mathrm{e}^{\frac{x}{3}}\,\mathrm{d}x$ **[3]**

The area bounded by the curve C, the tangent to the curve at P and the coordinate axes is shaded.

c Calculate the exact value of the shaded area. Show your working. **[5]**

14 $\mathrm{f}(x) = \dfrac{4}{\sqrt{x}(x-4)}$

Use the substitution $u = \sqrt{x}$ to find $\int \mathrm{f}(x)\,\mathrm{d}x$

Give your answer as a single logarithm in terms of x **[8]**

15 Use an appropriate substitution to find $\int x(2x-5)^4\,\mathrm{d}x$, give your answer in terms of x **[5]**

16 a Calculate $\int x^2 \sin x\,\mathrm{d}x$ **[4]**

b The area R is bounded by the curve $y = x^2 \sin x$, the x-axis and the lines $x = 0$ and $x = 2\pi$

Calculate the area of R. Show your working and give your answer in terms of π **[4]**

17 At time t minutes, the volume of water in a cylindrical tank is $V\,\mathrm{m}^3$. Water flows out of the tank at a rate proportional to the square root of V

a Show that the height of water in the tank satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{h}$$ **[4]**

b Find the general solution of this differential equation in the form $h = \mathrm{f}(t)$ **[3]**

The tank is 2 m tall and is initially full. It then takes 2 minutes to fully empty.

c Show that the particular solution to the differential equation is $h = 2\left(1 - \frac{1}{2}t\right)^2$ **[4]**