1 The area of a triangle is $\left(-1+3\sqrt{5}\right)$ cm². The height of the triangle is $\left(3+2\sqrt{5}\right)$ cm.

Show that the length of the base of the triangle is $\left(6-2\sqrt{5}\right)$ cm. **[4 marks]**

2 Find the solutions to the equation $2^{2x+1}-7(2^x)+6=0$. Show your working and give your answers to 3 significant figures where appropriate. **[4]**

3 $f(x)=x^2+(k+1)x+2$

a Find the range of values of k for which the equation $f(x)=0$ has distinct real roots. **[4]**

b Find the solutions of the equation $x^4-3x^2+2=0$. Show your working. **[3]**

4 Calculate the points of intersection between a circle with radius 5 and centre (1, 2) and a line that passes through the points (1, 3) and (–2, 6). Show your working. **[8]**

5 Find the range of values of x that satisfy both $2-2x-3x^2\geq 0$ and $4x+7>1$. Show your working. **[5]**

6 Solve the equation $5-\sin\theta-6\cos^2\theta=0$ for $0<\theta<360°$. Show your working. **[6]**

7 Solve the simultaneous equations $\quad e^{x+y}=3 \qquad 3x+2y=0$

Show your working and give each of your solutions as a single logarithm. **[5]**

8 $g(x)=6x^3-19x^2-12x+45$

a $y=g(x)$ and $y=0$ intersect at (3, 0) and at two other points. Calculate the remaining points of intersection, showing your working. **[5]**

b Sketch the curve $y=g(x)$, clearly labelling the points of intersection with the coordinate axes. **[3]**

c Calculate the area enclosed by the curve and the x-axis. Show your working. **[6]**

9 Find and classify all the stationary points of the curve with equation

$$y=\frac{1}{2}x^4-3x^3+2x^2+15x+1$$ **[8]**

Show your working.

10 The number of cases of a viral infection in a school with 2000 students after t days is given by $N=Ae^{kt}$. There are initially 2 cases of the infection and this number doubles after three days.

a Calculate the exact values of A and k **[4]**

b According to this model, how many days until a quarter of the students have been infected? Show your working. **[3]**

The number of cases of a second type of viral infection after t days is given by $M=Br^t$. There are initially 10 cases of this infection and after five days there are 15 cases.

c After how many days will the number of cases of the first infection overtake the number of the second infection? Show your working. **[7]**

d Sketch on the same axes the graphs of N and M against t for $t>0$ **[4]**

e How realistic do you think these models are? Explain your answer. **[3]**

11 $f(x)=6x^3-19x^2-51x-20$

a Show that $2x+1$ is a factor of $f(x)$ **[3]**

b Find all the solutions to $f(x)=0$, showing your working. **[3]**

12 Write each of these expressions in partial fractions.

a $\dfrac{3x+1}{(x+3)(2x+1)}$ **[4]** **b** $\dfrac{3x-5}{x^2-25}$ **[4]**

13 The function f is defined by $f: x \mapsto \dfrac{2x-14}{x^2-2x-3}+\dfrac{2}{x-3}, x>3$

a Show that f(*x*) can be written as $\dfrac{k}{x+1}$, where *k* is an integer to be found. **[4]**

b Write down the

i Domain of f(*x*) **ii** Range of f(*x*) **[3]**

c Find the inverse function, $f^{-1}(x)$ and state its domain. **[4]**

14 Given that $g(x)=x^2+3, x \in \mathbb{R}$ and $h(x)=\dfrac{3}{x-2}, x \neq 2$

a Write down hg(−2), **[3]**

b Solve the equation $gh(x)=12$. Show your working. **[4]**

c Is the range of gh(x) the same as the range of hg(x)? Explain how you know. **[3]**

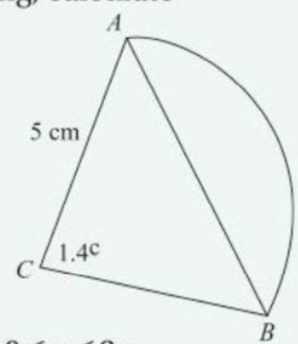
15 Work out the first four terms of the binomial expansion of $(1+2x)^{\frac{1}{4}}, |x|<\dfrac{1}{2}$, in ascending powers of *x* **[4]**

16 A sequence is defined by $u_{n+1}=2u_n+1,\ u_1=-2$. Showing your working, calculate

a u_2 **[2]** **b** $\sum_{1}^{5} u_r$ **[3]**

17 *AB* is the arc of a circle of radius 5 cm and centre *C* as shown.

The segment *S* is bounded by the arc and the line *AB*



a Calculate the area of *S* **[4]**

b Calculate the perimeter of *S* **[4]**

18 a Sketch each of these graphs on separate axes, for *x* in the range $0 \le x \le 2\pi$

i $y=2\sec x$ **ii** $y=\operatorname{cosec} 2x$ **[6]**

b Solve the equation $3\cot x+2=4$ for *x* in the range $0 \le x \le 360°$. Show your working. **[4]**

19 Showing your working, find the exact solutions to the equations.

a $2\arcsin x=\dfrac{\pi}{2}$ **[3]** **b** $\arctan 4x=\dfrac{\pi}{3}$ **[2]**

20 a Differentiate with respect to x

i $3\sin x$ **ii** $x\ln x$ **[3]**

Given that $f(x)=(2x+1)\cos x$

b Find the exact gradient of the curve $y=f(x)$ when $x=\frac{\pi}{6}$. Show your working. **[4]**

21 a Find the coordinates of the minimum point of the curve $y=xe^{2x}$. Show your working. **[5]**

b Explain how you know this is a minimum point. **[4]**

22 a Work out each of these integrals

i $\int \sin x\,dx$ **ii** $\int \frac{3}{x}\,dx$ **[3]**

b Calculate the exact value of the integral $\int_2^6 \frac{2}{x-1}\,dx$. Show your working and give your answer in its simplest form. **[4]**

23 Calculate the area bounded by the x-axis and the curve $y=\cos x$ for $0\le x\le \pi$. Show your working. **[4]**

24 $f(x)=x^3-6x-12$

a Show that the equation $f(x)=0$ has a root in the interval (3, 3.5). **[2]**

b Use the iterative formula $x_{n+1}=\sqrt{6+\frac{12}{x}}$, starting with $x_1=3$ to find x_2 and x_3 to 2 decimal places. **[2]**

c Prove that your value of x_3 is a solution to $f(x)=0$, correct to 2 decimal places. **[3]**

25 Use the trapezium rule with four strips to estimate the integral $\int_0^1 \cos^5 x\,dx$ to 3 significant figures. **[5]**

26 Prove by contradiction that if n^2 is odd then n is odd for all integers n **[5]**

27 Show that $\frac{2x^2+4x+3}{2x^2-x-1}$ can be written $A+\frac{B}{x-1}+\frac{C}{2x+1}$ where A, B and C are integers to be found. **[5]**

28 The function f is given by $f: x\to |3-2x|$

a Sketch the graph of $y=f(x)$. **[2]**

b How many solutions will there be to the equation $|3-2x|=x$? Explain how you know. **[2]**

c Solve the inequality $|3-2x|\ge x$, showing your working. **[4]**

29 $f(x)=\ln(3x+1),\ x>-\frac{1}{3}$

a Find the inverse $f^{-1}(x)$. **[3]**

b Sketch $y=f(x)$ and $y=f^{-1}(x)$ on the same axes. **[5]**

c Write down the range and domain of $f^{-1}(x)$. **[2]**

30 **a** Express $\dfrac{6x+10}{(x-1)(x+3)^2}$ in partial fractions. **[5]**

b Integrate $\dfrac{6x+10}{(x-1)(x+3)^2}$ with respect to x **[4]**

31 The points A and B have position vectors $12\mathbf{i}+7\mathbf{j}-5\mathbf{k}$ and $3\mathbf{i}-2\mathbf{j}-\mathbf{k}$ respectively.

Calculate the magnitude of the vector $\overrightarrow{AB}$. Show your working. **[4]**

32 A sequence is given by $x_{n+1}=(x_n)^2-2x_n$ where $x_1=1$

a Write down the value of x_2 and x_3 **[3]**

b Find an expression in terms of n for $\sum_{1}^{n} x_r$ **[4]**

33 The first term of a geometric series is 36 and the common ratio is $\frac{1}{3}$

a Find the difference between the second and third terms of the sequence. Show your working. **[3]**

b Calculate the difference between the sum to infinity and the sum of the first five terms of the series. Give your answer as a fraction. **[5]**

34 **a** Derive a formula for the sum of the first n terms of an arithmetic series with first term a and common difference d **[4]**

An arithmetic series has first term -3 and common difference 1.5.

The sum of the first n terms of this an arithmetic series is 63

b Find the value of n **[4]**

35 **a** Sketch the graph in part **i** for $-1 \le x \le 1$ and the graph in part **ii** for $-2 \le x \le 0$

i $y=\arccos x$ **ii** $y=2\arcsin(x+1)$ **[6]**

b State the range of each function in part **a**. **[2]**

c Write down the inverse of $f(x)=2\arcsin(x+1)$, $-2 \le x \le 0$ and state its domain. **[4]**

36 **a** Sketch the graph of $y=3\ln(x-1)$ and give the equation of any asymptotes. **[3]**

b Calculate the exact gradient of the curve at the point where $x=3$. Show your working. **[3]**

37 **a** Using a small angle approximation, show that $\sec 2x \approx \dfrac{1}{(1-2x^2)}$ **[4]**

b Hence, find the first three terms of the binomial expansion for $\sec 2x$ in ascending powers of x **[4]**

c Use your expansion to find an approximate value for $\sec(0.2)$ **[3]**