CM

A Level Maths Question Countdown

4 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self

Pure questions – standard

1 An arithmetic series has first term a and common difference d.

Prove that the sum of the first n terms of this series, S_n, is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.

2 Showing your working clearly, solve the equation

$$\cot 2\theta = \tan \theta$$

for $2\pi \leqslant \theta \leqslant 4\pi$.

Pure questions – problems

3 (a) On the same axes, sketch the curves with equation

(i) $y = 8 - 2x^2$

(ii) $y = \ln x$

showing clearly the coordinates of any points where the curves cross or meet the coordinate axes.

The equation

$$8 - 2x^2 = \ln x$$

has one real root α.

(b) Using your sketch, explain why $1 < \alpha < 2$

The iterative formula

$$x_{n+1} = \sqrt{4 - \frac{1}{2}\ln(x_n)}, \quad x_0 = 1$$

is used to find an approximation for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(d) Show that $\alpha = 1.917$ to three decimal places.

4 Show that

$$\int_{e}^{e^2} \frac{4}{x \ln x}\, dx = \ln 16$$

Pure questions – modelling

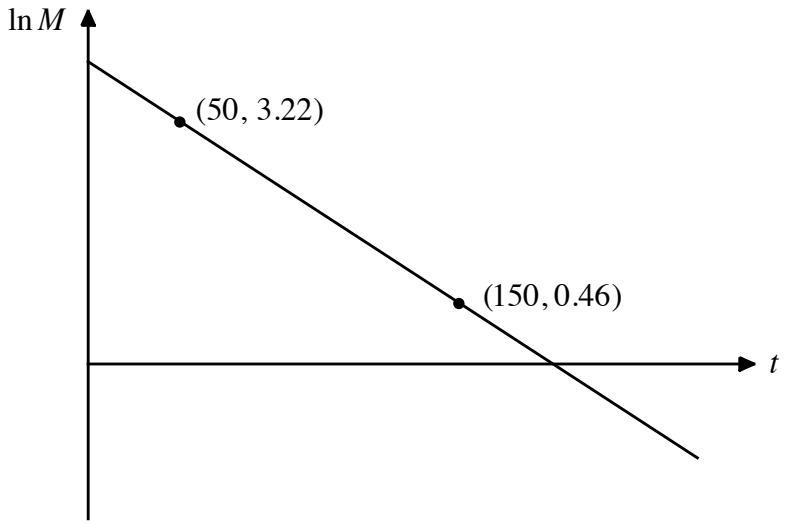
5 The mass of a radioactive substance at time t years is M grams.

Aidan models the rate of decrease of the mass of the substance to be proportional to the mass of the substance.

(a) Show that Aidan's model leads to the equation

$$M = a\mathrm{e}^{-kt}$$

where a and k are arbitrary positive constants.



Aidan finds archived data about the how the mass of the substance varies with time.

Using the data, he plots values of $\ln M$ against t. His results are shown in the diagram above.

(b) Explain why the results support Aidan's model.

(c) Hence express M in terms of t.

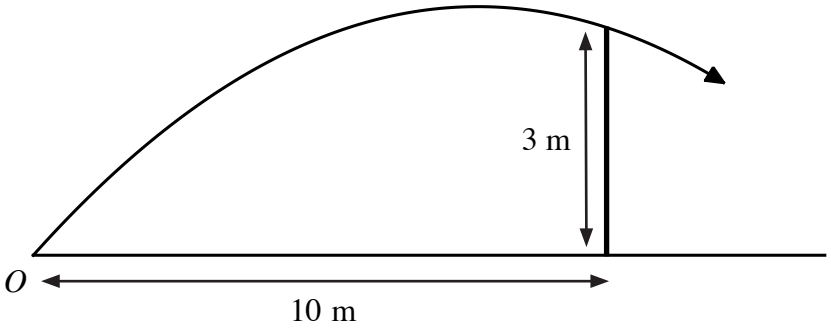
(d) Suggest **one** reason why the parameters in this model may not be suitable in modelling the decay of a different substance.

Applied questions – mechanics

6 A particle is projected from a point O with speed u at an angle of elevation θ above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x from O, its height above O is y.

(a) Show that

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$



A girl throws a ball from the ground towards a fence at an angle of elevation of 45° from a point O on the ground. The ball moves in a vertical plane that is perpendicular to the fence. The fence is 10 m away from the girl and the ball just passes over the top of the fence which is 3 m above the ground, as shown in the diagram.

The ball is modelled as a particle moving freely under the influence of gravity.

(b) Find the time taken for the ball to reach the fence from O.

(c) Calculate the speed and direction of the ball at the instant it passes over the fence.

(d) State how you have used the fact that the ball is a particle in your calculations.

Applied questions – statistics

7 Denise is investigating the cloud cover, C oktas, in Cambourne using the large data set.

(a) Using your knowledge of the large data set, explain why a normal distribution would be an inappropriate probability model for C.

Denise looks at the first 30 available data points for cloud cover in Cambourne in 1987. She finds that the proportion of days with cloud cover greater than 4 oktas is 0.8.

Denise believes that the proportion of days with cloud cover greater than 4 oktas in Cambourne 2015 has increased since 1987.

To test her claim, Denise uses a hypothesis test.

(b) Write down suitable hypotheses for Denise's hypothesis test.

She takes a random sample of 30 days in Cambourne in 2015. For these days, she finds that 27 of the 30 days have a cloud cover greater than 4 oktas.

(c) Assuming that C is binomially distributed, test Denise's claim at the 5% level of significance.

(d) Using your knowledge of the large data set, state one limitation of Denise's sampling