Statistical Distributions

Another treat of a topic for probability fans (like me), and your first glimpse of the binomial distribution. If you don't warm to the binomial distribution here, you'll get another chance with hypothesis testing in the next topic.

1 The discrete random variable X has the probability function shown below.

$$P(X = x) = \begin{cases} \frac{kx}{6} & \text{for } x = 1, 2, 3 \\ \frac{k(7-x)}{6} & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of k.

$k =$ .....

(2 marks)

b) Find $P(1 < X \leq 4)$.

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(2 marks)

A discrete random variable Y has the probability function $P(Y = y) = 0.2$ for $y = 1, 2, 3, 4, 5$.

c) State the name of the distribution of Y.

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(1 mark)

2 The number of points awarded to each contestant in a talent competition is modelled by the discrete random variable X with the following probability distribution:

x	0	1	2	3
$P(X = x)$	0.4	0.3	a	b

A contestant is twice as likely to be awarded 2 points as they are to be awarded 3 points.

By finding the values of a and b, calculate the probability that for two randomly chosen contestants, one scores 2 points and the other scores 3 points.

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(4 marks)

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3 In a game, a player tosses three fair coins. If three heads occur then the player wins 20p. If two heads occur then the player wins 10p. For any other outcome, the player wins nothing.

a) If X is the random variable 'amount won in pence', draw a table to show the probability distribution of X.

(3 marks)

The player pays 10p to play each game.

b) Use the probability distribution to find the probability that the player makes a profit over two games.

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(2 marks)

4 5% of chocolate bars made by a particular manufacturer contain a 'golden ticket'. A student buys 5 of the chocolate bars every week for 8 weeks.

The number of golden tickets he finds is represented by the random variable X.

a) State two necessary conditions for X to follow the binomial distribution B(40, 0.05).

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(2 marks)

Assuming that $X \sim B(40, 0.05)$:

b) Find $P(X > 1)$.

You can use the binomial tables or your calculator here.

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(2 marks)

c) Find the probability that more than 35 of the chocolate bars bought by the student do not contain a golden ticket.

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(2 marks)

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5 A particular model of car, the Dystopia, is prone to developing a rattle in the first year after being made. The probability of any particular Dystopia developing the rattle in its first year is 0.65.

A random sample of 20 one-year-old Dystopias is selected.

a) Find the probability that at least 12 but fewer than 15 of the cars rattle.

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(3 marks)

b) Find the probability that more than half of the cars rattle.

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(2 marks)

c) A further five random samples of Dystopias are tested. There are 20 cars in each sample. Find the probability that more than half of the cars in exactly three of these five samples rattle.

Define a new random variable that follows a binomial distribution with $n = 5$ and p = P(more than half of cars rattle).

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(3 marks)

6 An ice cream shop owner finds that, on 1st July, 880 out of the 1100 customers chose a sugar cone.

a) A random sample of 20 customers from 2nd July is selected. Use a binomial distribution and the data from the previous day to estimate the probability that exactly 12 of them chose a sugar cone.

Use the info to find p.

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(3 marks)

The owner claims that 42% of customers buy an ice cream with at least one scoop of chocolate ice cream.

b) Assuming that this claim is correct, and that the next 75 customers form a random sample of customers, find the probability that more than 30 of them choose at least one scoop of chocolate ice cream.

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(3 marks)

c) Comment on the validity of the binomial model you used in part b).

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(1 mark)

EXAM TIP

Remember, the binomial distribution is discrete, which means that $P(X < x)$ does not equal $P(X \leq x)$ — so you need to be extra careful with the inequality signs when you're finding probabilities. Make sure you're clued-up on how the binomial functions on your calculator work — you'll definitely need to use them if n or p aren't given in the binomial tables.

Score

35