1. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

(b) Find the speed of B immediately after hitting the wall.

(2)

The first collision between A and B occurred at a distance 4a from the wall. The balls collide again T seconds after the first collision.

(c) Show that $T = \frac{112a}{15u}$.

(6)

(Total 15 marks)

2. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e,

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision.

(Total 7 marks)

3. Particles A, B and C of masses $4m$, $3m$ and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u m s^{-1} and v m s^{-1} respectively, and collide directly.

As a result of the collision, A is brought to rest and B rebounds with speed kv m s^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Show that $u = 3v$.

(6)

(b) Find the value of k.

(2)

Immediately after the collision between A and B, particle C is projected with speed $2v$ m s^{-1} towards B so that B and C collide directly.

(c) Show that there is no further collision between A and B.

(4)
(Total 12 marks)

4. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$.

(4)

The collision between *P* and *Q* takes place at the point *A*. After the collision *Q* hits a smooth fixed vertical wall which is at right-angles to the direction of motion of *Q*. The distance from *A* to the wall is *d*.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when *Q* hits the wall.

(4)

Particle *Q* rebounds from the wall and moves so as to collide directly with particle *P* at the point *B*. Given that the coefficient of restitution between *Q* and the wall is $\frac{1}{5}$,

(d) find, in terms of *d*, the distance of the point *B* from the wall.

(4)
(Total 17 marks)

5. Two small smooth spheres *A* and *B* have equal radii. The mass of *A* is $2m$ kg and the mass of *B* is m kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of *A* is $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1} and the velocity of *B* is $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of *A* is $(\mathbf{i} - 3\mathbf{j})$ m s^{-1}. Find the speed of *B* immediately after the collision.

(Total 5 marks)

6. A particle *A* of mass $2m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle collides directly with a particle *B* of mass m moving with speed $2u$ in the opposite direction to *A*. Immediately after the collision the speed of *B* is $\frac{8}{3}u$ and the direction of motion of *B* is reversed.

(a) Calculate the coefficient of restitution between *A* and *B*.

(6)

(b) Show that the kinetic energy lost in the collision is $7mu^2$.

(3)

After the collision B strikes a fixed vertical wall that is perpendicular to the direction of motion of B. The magnitude of the impulse of the wall on B is $\frac{14}{3}mu$.

(c) Calculate the coefficient of restitution between B and the wall.

(4)
(Total 13 marks)

7. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal table. The particle P collides with a particle Q of mass $2m$ moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q after the collision is $\frac{1}{5}u(9e + 4)$.

(5)

As a result of the collision, the direction of motion of P is reversed.

(b) Find the range of possible values of e.

(5)

Given that the magnitude of the impulse of P on Q is $\frac{32}{5}mu$,

(c) find the value of e.

(4)
(Total 14 marks)

8. A smooth sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another smooth sphere B of mass $3m$, which is at rest on the table. The coefficient of restitution between A and B is e. The spheres have the same radius and are modelled as particles.

(a) Show that the speed of B immediately after the collision is $\frac{1}{4}(1 + e)u$.

(5)

(b) Find the speed of A immediately after the collision.

(2)

Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6}mu^2$.

(c) Find the value of e.

(6)

(d) Hence show that A is at rest after the collision.

(1)
(Total 14 marks)

9. A uniform sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another uniform sphere B of mass $2m$ which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is e. The direction of motion of A is unchanged by the collision.

(a) Find the speeds of A and B immediately after the collision.

(7)

(b) Find the range of possible values of e.

(2)

After being struck by A, the sphere B collides directly with another sphere C, of mass $4m$ and of the same size as B. The sphere C is at rest on the table immediately before being struck by B. The coefficient of restitution between B and C is also e.

(c) Show that, after B has struck C, there will be a further collision between A and B.

(6)
(Total 15 marks)

10. A smooth sphere P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

(a) Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively.

(7)

After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

(b) Find the value of e.

(7)

(c) Explain why there must be a third collision between P and Q.

(1)
(Total 15 marks)

11. A smooth sphere is moving with speed U in a straight line on a smooth horizontal plane. It strikes a fixed smooth vertical wall at right angles. The coefficient of restitution between the sphere and the wall is $\frac{1}{2}$.

Find the fraction of the kinetic energy of the sphere that is lost as a result of the impact.

(Total 5 marks)

12. A smooth sphere S of mass m is moving with speed u on a smooth horizontal plane. The sphere S collides with another smooth sphere T, of equal radius to S but of mass km, moving in the same straight line and in the same direction with speed λu, $0 < \lambda < \frac{1}{2}$. The coefficient of restitution between S and T is e.

Given that S is brought to rest by the impact,

(a) show that $e = \dfrac{1 + k\lambda}{k(1 - \lambda)}$.

(6)

(b) Deduce that $k > 1$.

(3)
(Total 9 marks)

1. (a) (i)

Con. of Mom: $3mu - mu = 3mv + mw$

$2u = 3v + w$ (1) M1# A1

N.L.R: $\frac{1}{2}(u+u) = w - v$ M1# A1

$u = w - v$ (2)

(1) – (2) $u = 4v$ DM1#

$v = \frac{1}{4}u$ A1

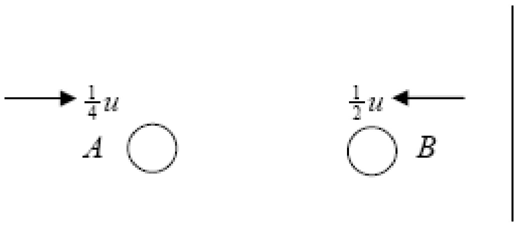
(ii) In (2) $u = w - \frac{1}{4}u$

$w = \frac{5}{4}u$ A1 7

(b) B to wall: N.L.R: $\frac{5}{4}u \times \frac{2}{5} = V$ M1

$V = \frac{1}{2}u$ A1ft 2

(c)



B to wall: $\text{time} = 4a \div \dfrac{5}{4}u = \dfrac{16a}{5u}$ B1ft

Dist. Travelled by $A = \dfrac{1}{4}u \times \dfrac{16a}{5u} = \dfrac{4}{5}a$ B1ft

In t secs, A travels $\dfrac{1}{4}ut$, B travels $\dfrac{1}{2}ut$

Collide when speed of approach $= \dfrac{1}{2}ut + \dfrac{1}{4}ut$, distance to cover = M1$

$4a - \dfrac{4}{5}a$

$\therefore\ t = \dfrac{4a - \frac{4}{5}a}{\frac{3}{4}u} = \dfrac{16a}{5} \times \dfrac{4}{3u} = \dfrac{64a}{15u}$ DM1$ A1

Total time $= \dfrac{16a}{5u} + \dfrac{64a}{15u} = \dfrac{112a}{15u}$ * A1 6

[15]

2.

$\xrightarrow{2u}$ $\xleftarrow{u}$

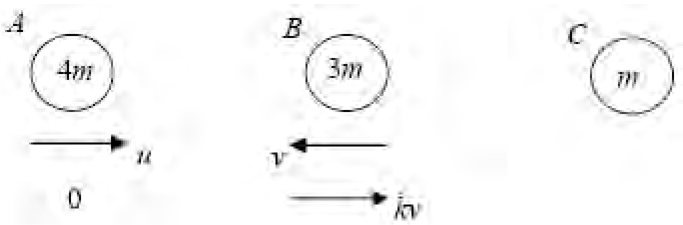
$\xrightarrow{2m}$ $\xrightarrow{m}$

v_1 v_2

CLM: $4mu - mu = 2mv_1 + mv_2$	M1 A1
i.e. $3u = 2v_1 + v_2$	
NIL: $3eu = -v_1 + v_2$	M1 A1
$v_1 = u(1 - e)$	DM1 A1
$v_2 = u(1 + 2e)$	A1

[7]

3. (a)



Conservation of momentum:	$4mu - 3mv = 3mkv$	M1A1	
Impact law:	$kv = \frac{3}{4}(u+v)$	M1A1	
Eliminate k:	$4mu - 3mv = 3m \times \frac{3}{4}(u+v)$	DM1	
	$u = 3v$ (Answer given)	A1	6

(b) $kv = \frac{3}{4}(3v+v), k = 3$ M1,A1 2

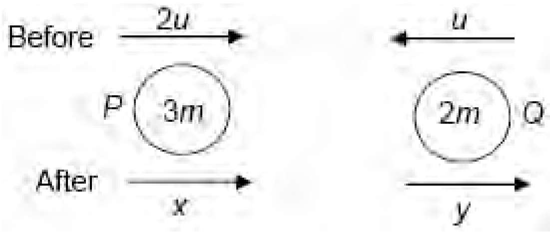
(c) Impact law: $(kv + 2v)\, e = v_C - v_B$ $(5ve = v_C - v_B)$ B1

Conservation of momentum : $3\times kv - 1 \times 2v = 3v_B + v_C$ $(7v = 3v_B + v_c)$ B1

Eliminate $v_C : v_B = \frac{v}{4}(7-5e) > 0$ hence no further collision with *A*. M1 A1 4

[12]

4. (a)



	Correct use of NEL	M1 *	
	$y - x = e(2u + u)$ o.e.	A1	
	CLM ($\rightarrow$): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$)	B1	
	Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$)	d * M1	
	Hence, speed of Q = $\underline{\tfrac{1}{5}(9e+4)u}$ **AG**	A1 cso	5
(b)	$x = y - 3eu = \frac{1}{5}(9e+4)u - 3eu$	M1 *	
	Hence, speed $\text{P} = \frac{1}{5}(4-6e)u = \frac{2u}{5}(2-3e)$ o.e.	A1	
	$x = \frac{1}{2}u = \frac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	d * M1	
	gives, $\boldsymbol{e}$ = $\tfrac{3}{12} \Rightarrow$ $\underline{\mathbf{e} = \tfrac{1}{4}}$ **AG**	A1	
	Or		
	Using NEL correctly with given speeds of P and Q	M1 *	
	$3eu = \tfrac{1}{5}(9e+4)u - \tfrac{1}{2}u$	A1	
	$3eu = \tfrac{9}{5}eu + \tfrac{4}{5}u - \tfrac{1}{2}u$, $3e - \tfrac{9}{5}e = \tfrac{4}{5} - \tfrac{1}{2}$ & solve for e	d * M1	
	$\tfrac{6}{5}e = \tfrac{3}{10} \Rightarrow e = \tfrac{15}{60} \Rightarrow e = \tfrac{1}{4}$.	A1	4
(c)	Time taken by Q from A to the wall = $\frac{d}{\underline{y}} = \left\{\frac{4d}{5u}\right\}$	M1^{+}	
	Distance moved by P in this time = $\frac{u}{2} \times \frac{d}{y} \left(= \frac{u}{2}\left(\frac{4d}{5u}\right) = \frac{2}{5}d\right)$	A1	
	Distance of P from wall = $d - \times\left(\frac{d}{y}\right); = d - \tfrac{2}{5}d = \underline{\tfrac{3}{5}d}$ **AG**	d^{+}M1;	
		A1 cso	
	or		
	Ratio speed P:speed Q = x:y = $\frac{1}{2}u : \frac{1}{5}(\frac{9}{4}+4)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$	M1^{+}	
	So if Q moves a distance d, P will move a distance $\tfrac{2}{5}d$	A1	
	Distance of P from wall = $d - \tfrac{2}{5}d; = \underline{\tfrac{3}{5}d}$ **AG** cso	d^{+}M1;A1	4

(d) After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

Time for P, $T_{AB} = \dfrac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \dfrac{x}{\frac{1}{4}u}$ from their y B1ft

Hence $T_{AB} = T_{WB} \Rightarrow \dfrac{\frac{3d}{5} - x}{\frac{1}{2}u} = \dfrac{x}{\frac{1}{4}u}$ M1

gives, $2(\frac{3d}{5} - x) = 4x \Rightarrow \frac{3d}{5} - x = 2x, 3x = \boldsymbol{\frac{3d}{5}} \Rightarrow x = \underline{\frac{1}{5}d}$ A1 cao

or

After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ B1 ft
from their y

Distance of B from wall = $\dfrac{1}{3} \times \dfrac{3d}{5}; = \dfrac{d}{5}$ their $\dfrac{1}{2+1}$ M1; A1 4

2nd or

After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$

Time from wall to 2nd collision = $\dfrac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y B1ft

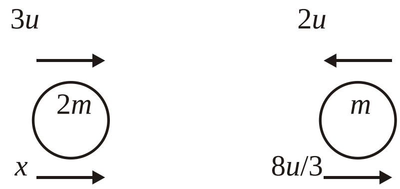
Distance of B from
wall = (their speed)x(their time) = $\dfrac{u}{4} \times \dfrac{4d}{5u}; = \underline{\frac{1}{5}d}$ M1; A1 4

[17]

5. $2m(2\mathbf{i} - 2\mathbf{j}) + m(-3\mathbf{i} - \mathbf{j}) = 2m(\mathbf{i} - 3\mathbf{j}) + m\boldsymbol{v}$ M1A1
$(\mathbf{i} - 5\mathbf{j}) = (2\mathbf{i} - 6\mathbf{j}) + \mathbf{v}$ M1A1
$(-\mathbf{i} + \mathbf{j}) = \mathbf{v}$ A1
$|\mathbf{v}| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ m s^{-1} cwo DM1A1 5

[5]

(a)



LM $\quad 6mu - 2mu = 2mx + \frac{8}{3}mu$ M1 A1

$\left(x = \frac{2}{3}u\right)$

NEL $\quad \frac{8}{3}u - x = 5ue$ M1 A1

Solving to $e = \frac{2}{5}$ M1 A1 6

(b) Initial K.E. = $\frac{1}{2} \times 2m(3u)^2 + \frac{1}{2} \times m(2u)^2 = 11\,mu^2$

Final K.E. = $\frac{1}{2} \times 2m\left(\frac{2}{3}u\right)^2 + \frac{1}{2} \times m\left(\frac{8}{3}u\right)^2 = 4mu^2$ both M1

Change in K.E. = $7mu^2$ M1 Subtracting and simplifying to kmu^2 A1cso M1 A1 3

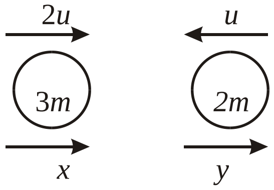
(c) $m\left(\frac{8}{3}u + v\right) = \frac{14}{3}mu$ M1 A1

($v = 2u$)

$e = \frac{2}{8/3} = \frac{3}{4}$ M1 A1 4

[13]

6.



(a) LM $6mu - 2mu = 3mx + 2my$ M1 A1

NEL $\quad y - x = 3eu$ B1

Solving to $y = \frac{1}{5}u(9e + 4)$ (*) cso M1 A1 5

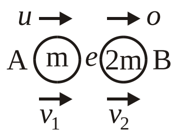
	(b)	Solving to $x = \frac{2}{5}u(2 - 3e)$	M1 A1	
		oe		
		$x < 0 \Rightarrow e > \frac{2}{3}$	M1 A1	
		$\frac{2}{3} < e \leq 1$	A1ft	5
		ft their e for glb		
	(c)	$2m[\frac{1}{5}u(9e + 4) + u] = \frac{32}{5}mu$	M1 A1	
		Solving to $e = \frac{7}{9}$	M1 A1	4
		awrt 0.78		

[14]

7.	(a)	$u \rightarrow$ $\rightarrow 0$ CLM: $mu = mv_1 + 3mv_2$	B1	
		m $3m$ NIL: $eu = -v_1 + v_2$	M1 A1	
		$v_1 \rightarrow$ $v_2 \rightarrow$ solving,	**dep**. M1	
		$v_2 = \frac{u}{4}(1 + e)$*	A1	5
	(b)	Solving for v_1; $\left\lvert \frac{u}{4}(1-3e) \right\rvert$	M1 A1	2
	(c)	$\frac{1}{2}m\frac{u^2}{16}(1 - 3e)^2 + \frac{1}{2}3m\frac{u^2}{16}(1 + e)^2 = \frac{1}{6}mu^2$	M1 A1 f.t. A1	
		$e^2 = \frac{1}{9}$	**dep**. M1 A1	
		$e = \frac{1}{3}$	A1	6
	(d)	$v_1 = \frac{u}{4}(1 - 3 \times \frac{1}{3}) = 0 \Rightarrow$ at rest.	A1 c.s.o.	1

[14]

8. (a)



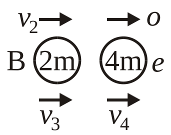
$mu = mv_1 + 2mv_2$ M1 A1

$eu = -v_1 + v_2$ M1 A1

$v_1 = \frac{u}{3}(1 - 2e)$; $v_2 = \frac{u}{3}(1 + e)$ M1 A1 A1 7

(b) $v_1 > 0 \Rightarrow \frac{u}{3}(1 - 2e) > 0 \Rightarrow e < \frac{1}{2}$ M1 A1 c.s.o. 2

(c)



$2mv_2 = 2mv_3 + 4mv_4$ M1

$ev_2 = -v_3 + v_4$

$v_3 = \frac{v_2}{3}(1 - 2e) = \frac{u}{9}(1 - 2e)(1 + e)$ M1 A1

Further collision if $v_1 > v_3$

i.e. if $\frac{u}{3}(1 - 2e) > \frac{u}{9}(1 - 2e)(1 + e)$ M1

i.e. if $3 > 1 + e$ (as $(1 - 2e) > 0$)

i.e. if $2 > e$ M1

which is always true, so further collision occurs A1 cso 6

[15]

9. (a) L.M. $2u = 2x + y$ M1 A1

NEL $y - x = \frac{1}{3}u$ M1 A1

Solving to $x = \frac{5}{9}u$ (*) M1 A1

$y = \frac{8}{9}u$ (*) A1 7

(b)	$(\pm)\ \frac{8}{9}eu$	B1	
	L.M $\frac{10}{9}u - \frac{8}{9}eu = w$	M1 A1	
	NEL $w = \frac{1}{3}\left(\frac{5}{9}u + \frac{8}{9}eu\right)$	M1 A1	
	Solving to $e = \frac{25}{32}$	M1 A1	7
	accept 0.7812s		
(c)	Q still has velocity and will *bounce back* from wall colliding with *stationary P*.	B1	1

[15]

10.	$v = \frac{1}{2}u$	B1	
	KE loss = $\frac{1}{2}m\left(u^2 - \left(\frac{1}{2}u\right)^2\right)$	M1	
	$= \frac{3mu^2}{8}$	A1	
	$\therefore$ fraction of KE lost = $\frac{3mu^2}{8} \div \frac{1}{2}mu^2 = \frac{3}{4}$	M1 A1	5

[5]

11. (a)

$u \rightarrow$ $\rightarrow \lambda u$

S m km T

$\rightarrow$ $\rightarrow$

0 v

$mu + km\lambda u = kmv$		
$\frac{u}{k}(1 + k\lambda) = v$	M1 A1	
$v = e(u - \lambda u) = eu(1 - \lambda)$	M1 A1	
$\Rightarrow \frac{u}{k}(1 + k\lambda) = eu(1 - \lambda)$	M1	
$\Rightarrow \frac{(1+k\lambda)}{k(1-\lambda)} = e$ (T)	A1	6

(b)	$\dfrac{1+k\lambda}{k(1-\lambda)} \leq 1 \Rightarrow \quad 1 + k\lambda \quad \leq \quad k(1-\lambda)$	M1	
	$\Rightarrow \dfrac{1}{1-2\lambda} \leq k$	A1	
	since $0 < \lambda < \frac{1}{2}$, $k > 1$ (T)	A1	3

[9]

1. There were also a significant number of fully correct answers to this question. Most candidates completed parts (a) and (b) well but part (c) proved to be rather more demanding.

For part (a) the majority of candidates understood and applied the conservation of linear momentum and the law of restitution correctly. The equations were usually consistent despite the occasional lack of a clear diagram, and only a very small minority got the restitution equation the wrong way round.

However, subsequent errors in the speed of *A* and *B* were common – these arose from simple sign errors in the initial equations or more commonly from minor processing errors. These basic errors can be costly – unexpected outcomes should be checked carefully to avoid continuing to work with unrealistic situations. A few candidates surprisingly failed to substitute ½ in for e and then struggled through with their answers for the rest of the question.

Full marks were usually scored for part (b) with only a minority of candidates with method errors in the use of the restitution equation. The question asked for the speed of B after the collision, so the final answer should have been positive, which was not always the case.

There were a wide variety of approaches to part (c). It was pleasing to see that some candidates could produce the given expression fluently with their methods clearly laid out. The most successful approach involved finding the separation of the two particles as *B* impacted with the wall and then to use the relative velocity to find the time taken to cover this separation. Some used the ratio of distances travelled or set up an equation in *T*.

Many could find the time to *B* hitting the wall and the distance travelled by *A* in this time, but got no further, having run out of time or having no idea how to proceed further.

A few solutions went off in entirely the wrong direction, either by thinking that another collision was needed (considering CLM and NEL again) or by attempting to use methods which implied non-zero acceleration. A worrying handful did not use the correct relationship between distance speed and time (e.g time = distance × speed was seen).

Expressions involving *a* and *u* were often badly written and these symbols would become interchanged during the rearrangement and simplification of terms. For example, a fractional term such as $\frac{5}{4}u$ was written without due care so that the *u* migrated to the bottom of the fraction during manipulation. Clear layout and a description of the symbols are vital in this kind of question to avoid these careless errors and to help examiners navigate through a candidates' work.

The given answer was helpful to some candidates who made a false start - realising their error they would often produce a better or correct solution. However, candidates working with incorrect values from part (a) were often misled into altering work displaying correct method in an attempt to derive the given answer. There were also some who tried to fudge incorrect processes to achieve the given answer.

DM1$ A1

Total time $= \frac{16a}{5u} + \frac{64a}{15u} = \frac{112a}{15u}$ **26.** This question was very well answered by the majority of candidates. The momentum and impact equations were often correct - the most common error was lack of consistency in signs between their equations. A surprising number of candidates did not draw a diagram which possibly made it more difficult to avoid these sign errors. Only a small number of candidates quoted the impact law the wrong way round.

A significant number of candidates had *P* going to the left after the collision, obtaining a velocity of $u(e - 1)$. However, they almost always failed to realise that this was a negative answer, ignoring the fact that the question had asked for speed.
A number of candidates, who started with two correct equations, went on to lose marks at the end due to algebraic errors in solving the simultaneous equations.

2. Parts (a) and (b) were tackled with confidence by most candidates although a few were not sufficiently careful with signs and/or had long-winded algebraic manipulation to achieve the given result. CLM and the impact law were generally used correctly.

Part (c) proved to be more challenging and differentiated between the stronger candidates and those with confused concepts. No mention of e in the question caused some to ignore it or, more commonly, to assume the previous value. There were various arguments used to justify their final statement but, encouragingly, the range for e seemed to be understood.

3. Candidates made a confident start to this question, but in parts (a) and (b), poor algebraic skills and the lack of a clear diagram with the directions marked on it hampered weaker candidates' attempts to set up correct and consistent (or even physically possible) equations. The direction of *P* after impact was not given and those candidates who took its direction as reversed ran into problems when finding the value of e. Many realised that they had chosen the wrong direction and went on to answer part (b) correctly but some did not give an adequate explanation for a change of sign for their velocity of *P*. Algebraic and sign errors were common, and not helped by candidates' determination to reach the given answers.

Parts (c) and (d) caused the most problems. They could be answered using a wide variety of methods, some more formal than others. Many good solutions were seen but unclear reasoning and methods marred several attempts. Too many solutions were sloppy, with u or d appearing and disappearing through the working. A few words describing what was being calculated or expressed at each stage would have helped the clarity of solutions greatly. Students need to be reminded yet again that all necessary steps need to be shown when reaching a given answer.

Too many simply stated the answer $\frac{3d}{5}$ without the explanation to support it.

4. Many candidates failed to recognise this as one of the most straightforward questions they are ever likely to meet at this level. Those candidates who did recognise the need for a simple application of conservation of momentum, using the vectors given, were few and far between. It was more common for candidates to apply conservation of momentum in the **i** and **j** directions separately, and such candidates generally did so successfully. Many candidates wanted this to be the more usual style of question, where they could work parallel and perpendicular to the line of centres of the two spheres. Candidates who tried to find the line of centres were generally unsuccessful. Some assumed it was in either the **i** or the **j** direction and made no headway as a result. A large number of candidates were content to regard the velocity as the speed and did not attempt to find the magnitude of their velocity.

5. The first part was very well answered with the odd error of incorrect signs. It was pleasing to note that only a few gave the restitution equation the wrong way round. Most of the errors here occurred in parts (b) and (c). There appeared to be some confusion over the fact that the KE of every particle before and after the impact was required and that these needed to be added and subtracted correctly in order to get a positive loss of KE. Some candidates missed out the KE of one of the particles and many subtracted to get a negative KE. In part (c) the impulse equation was often incorrect, with an incorrect sign in the velocity term. This led to a value of e >1. Some candidates realised that this was wrong and corrected their equation but others changed the final equation but failed to change the signs in their original impulse equation and were unable to score full marks.

6. In part (a), nearly all candidates could obtain a pair of equations using conservation of linear momentum and Newton's law of restitution. The printed answer usually helped those who had made sign errors to correct them. Part (b) proved more difficult. The majority of candidates first found the velocity of *P*. A few, on recognising that the direction of *P* differed from the one they had used in part (a), reversed the direction of *P* and started all over again rather than correctly interpreting the results they already had. In doing so they sometimes confused their two sets of working. Those who had found the velocity of *P* often gave the inequality the wrong way round or produced fallacious work when solving the inequality. Another error, frequently seen, was to produce an inequality from the incorrect reasoning that the speed of one particle had to be greater than the other. The fully correct range $\frac{2}{3} < e \leq 1$ was rarely given. One neat, although equivalent, method of solution seen was to say that the velocity of separation had to be greater than the velocity of *Q*. This gives $3eu > \frac{1}{5}u\left(9e+4\right)$ without the necessity of finding the velocity of *P*. Part (c) is demanding in its algebraic requirements and the signs were difficult for many to sort out. However many completely correct solutions were seen and the general standard of algebraic manipulation was good.

7. Part (a) was successfully completed by most of the candidates who attempted the question, although some thought that the two particles had the same final speed. In part (b), most were able to obtain an expression for a velocity but the significance of the word 'speed' was not understood and there were perhaps only one or two correct answers from the entire candidature. In the third part most candidates were unsuccessful in obtaining e=1/3 due to either forgetting that the mass of *B* was 3*m* or their inability to accurately expand and collect terms from two squared brackets.

Others added the velocities of *A* and *B* together and squared the total and a significant number just "cancelled" the squares off the three expressions in the energy equation reducing it to $u = v_1 + 3v_2$. Some may have misread the question and assumed that the total KE before and after the collision was 1/6$m\,u^2$ gaining an extra KE term. Some candidates realised from their answer to (b) that e should come out to be 1/3 and simply fiddled their working. Even some of those successful in (c) did not justify or show by substitution that the velocity of *A* was zero, merely stating the fact without any reference even to e=1/3.

8. (a) The method here was generally well known and the momentum equation was usually correct although the NIL equation sometimes had sign errors. Manipulation of the equations was generally poor.

(b) This was rarely correct – a common error was to compare the two velocities. Many answers were unsupported and got no credit.

(c) A significant number were able to write down attempts at the two equations but only a few were able to progress further as the algebra became more complex. A tiny number realised that the second impact was essentially the same as the first and were able to write down the final velocities without any further working. A full and convincing argument for this part was very rare.

9. The most notable characteristic about the majority of candidates' responses to this question was the number of completely correct solutions seen. The algebra in part (a) was quite straightforward but the same was definitely not true of part (b) and yet many could negotiate the minimum of four variables needed in this question and obtain the correct exact answer $e = \frac{25}{32}$. The principles needed to solve part (a) were well understood. When errors were made in part (b), these usually arose from not realising that there were two separate impacts to consider, one between *Q* and the wall, with an unknown *e* and a second impact between *P* and *Q*, in which $e = \frac{1}{3}$. Errors of sign were also seen, often resulting in a pair of incompatible equations for linear momentum and Newton's Experimental Law. In part (c), candidates were expected to explain that there would be a second impact between *Q* and the wall, which would result in a further impact between *Q* and the stationary *P*.

10. No Report available for this question.

11. No Report available for this question.