Exam Questions – Modelling in Mechanics (45 mins)

1 The motion of a cricket ball after it is hit until it lands on the cricket pitch can be modelled using the equation $h = \frac{1}{10}(24x - 3x^2)$, where h m is the vertical height of the ball above the cricket pitch and x m is the horizontal distance from where it was hit. Find:

a the vertical height of the ball when it is at a horizontal distance of 2 m from where it was hit

b the two horizontal distances for which the height of the ball was 2.1 m.

Given that the model is valid from when the ball is hit to when it lands on the cricket pitch:

c find the values of x for which the model is valid

d work out the maximum height of the cricket ball.

a) $h = \frac{1}{10}(24(2) - 3(2)^2)$

$h = 3.6$

b) $2.1 = \frac{1}{10}(24x - 3x^2)$

$21 = 24x - 3x^2$

$3x^2 - 24x + 21 = 0$

$3(x - 8x + \ ) = 0$

$3(x - 7)(x - 1) = 0$

$x = 7$ or 1

c) $0 \leq x \leq 8$

d) $x = 4 \quad h = 4.8$

2 Make a list of the assumptions you might make to create simple models of the following:

a The motion of a man skiing down a snow-covered slope.

b The motion of a yo-yo on a string.

In each case, describe the effects of the modelling assumptions.

a) model skier as a particle
ignore air resistance
snow is smooth (no friction)

b) model yo yo as a particle
ignore air resistance
string is light and inextensible

3 Convert to SI units:

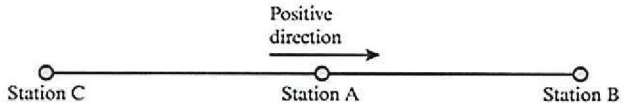
a 2.5 km per minute **b** 0.6 kg cm^{-2} **c** 1.2×10^3 g cm^{-3}

a) $\frac{2.5 \times 1000}{60} = 41.7$ ms^{-1}

b) $0.6 \times 100^2 = 6000$ kg m^{-2}

c) $\left(\frac{1.2 \times 10^3}{1 \div 100^3}\right) \div 1000 = 1.2 \times 10^6$ kg m^{-3}

4 A train engine pulling a truck starts at station A then travels in a straight line to station B. It then moves back from station B to station A and on to station C as shown in the diagram.



What is the sign of the velocity and displacement on the journey from:

a station A to station B **b** station B to station A **c** station A to station C?

a) Velocity positive displacement positive

b) Velocity negative displacement positive

c) Velocity negative displacement negative

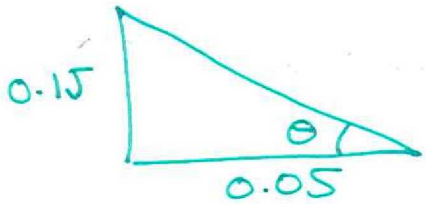
5 The acceleration of a boat is given by $\mathbf{a} = -0.05\mathbf{i} + 0.15\mathbf{j}\ \text{m s}^{-2}$. Find:

a the magnitude of the acceleration

b the angle the direction of the acceleration vector makes with the unit vector $\mathbf{i}$.

a) $|a| = \sqrt{(-0.05)^2 + (0.15)^2} = 0.158$

b)



$\theta = 71.6$

required angle $= 180 - 71.6$

$= 108°$

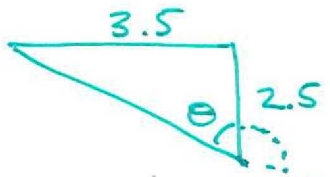
6 The velocity of a toy car is given by $\mathbf{v} = 3.5\mathbf{i} - 2.5\mathbf{j}\ \text{m s}^{-1}$. Find:

a the speed of the toy car

b the angle the direction of motion of the toy car makes with the unit vector $\mathbf{j}$.

a) $|v| = \sqrt{3.5^2 + 2.5^2} = 4.30\ \text{m s}^{-1}$

b)



$\theta = 54.5°$

Required angle $= 180 - 54.5$

$= 126°$

7 A plane flies from P to Q and then from Q to R.

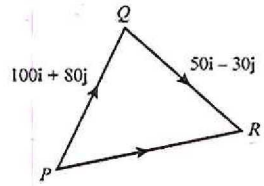
The displacement from P to Q is $100\mathbf{i} + 80\mathbf{j}$ m.

The displacement from Q to R is $50\mathbf{i} - 30\mathbf{j}$ m.

a Find the magnitude of the displacement from P to R.

b Find the total distance the plane has travelled in getting from P to R.

c Find the angle the vector $\overrightarrow{PQ}$ makes with the unit vector $\mathbf{j}$.



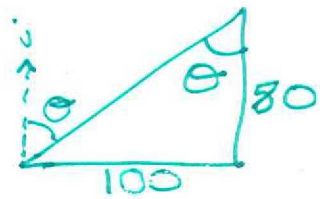
a) $\overrightarrow{PR} = 100\mathbf{i} + 80\mathbf{j} + 50\mathbf{i} - 30\mathbf{j}$

$= 150\mathbf{i} + 50\mathbf{j}$

$|PR| = \sqrt{150^2 + 50^2} = \sqrt{25000} = 158$

b) $|PQ| = 128$ $|QR| = 58.3$ $|PQ| + |QR| = 186$

c)



$\theta = 51.3°$