FP1 – Matrix Algebra Questions ANSWERS (63 marks)

Jan 2013

4.	(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **(1)**
	(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 **(1)**
	(c) $\mathbf{R} = \mathbf{QP}$	B1 **(1)**
	(d) $\mathrm{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao **(2)**
	(e) Reflection in the y axis	B1 B1 **(2)** [7]
Notes	(a) and (b) Signs must be clear for B marks. (c) Accept **QP** or their 2x2 matrices in the correct order only for B1. (d) M for their **QP** where answer involves ±1 and 0 in a 2x2 matrix, A for correct answer only. (e) First B for Reflection, Second B for 'y axis' or 'x=0'. Must be single transformation. Ignore any superfluous information.	

6.	(a) Determinant: $2 - 3a = 0$ and solve for $a =$	M1
	So $a = \frac{2}{3}$ or equivalent	A1
		(2)
	(b) Determinant: $(1\times 2)-(3\times -1)=5$ (Δ)	
	$Y^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}$ $\left[= \begin{pmatrix} 0.4 & 0.2 \\ -0.6 & 0.2 \end{pmatrix}\right]$	M1A1
		(2)
	(c) $\frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1-\lambda \\ 7\lambda - 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2-2\lambda+7\lambda-2 \\ -3+3\lambda+7\lambda-2 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda - 1 \end{pmatrix}$	M1depM1A1 A1
		(4) **[8]**
	<u>Alternative method for (c)</u>	
	$\begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-\lambda \\ 7\lambda - 2 \end{pmatrix}$ so $x - y = 1 - \lambda$ and $3x + 2y = 7\lambda - 2$	M1M1
	Solve to give $x = \lambda$ and $y = 2\lambda - 1$	A1A1
Notes	(b) M for $\frac{1}{\text{their det}}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}$	
	(c) First M for their $\mathbf{Y}^{-1}\mathbf{B}$ in correct order with **B** written as a 2x1 matrix, second M dependent on first for attempt at multiplying their matrices resulting in a 2x1 matrix, first A for λ, second A for $2\lambda - 1$	
	Alternative for (c) First M to obtain two linear equations in x, y, λ Second M for attempting to solve for x or y in terms of λ	

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2. (a)	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 3+1+0 & 3+2-3 \\ 4+5+0 & 4+10-5 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct (unsimplified) elements in a dimensionally correct matrix. A 2x2 matrix with a number or a calculation at each corner.	M1
	$= \begin{pmatrix} 4 & 2 \\ 9 & 9 \end{pmatrix}$	Correct answer	A1
	A correct answer with no working can score both marks		
			[2]
(b)	$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$, where k is a constant,		
	$\mathbf{C} + \mathbf{D} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix} = \begin{pmatrix} 8 & 2k+2 \\ 12 & 6+k \end{pmatrix}$	An attempt to add C to D. Can be implied by two out of four correct (unsimplified) elements in a dimensionally correct matrix.	M1
	E does not have an inverse $\Rightarrow$ $\det \mathbf{E} = 0$.		
	$8(6+k) - 12(2k+2)$	Applies "$ad - bc$" to **E** where **E** is a 2x2 matrix.	M1
	$8(6+k) - 12(2k+2) = 0$	States or applies $\det(\mathbf{E}) = 0$ where $\det(\mathbf{E}) = ad - bc$ or $ad + bc$ only and **E** is a 2x2 matrix.	M1
	Note $8(6+k) - 12(2k+2) = 0$ or $8(6+k) = 12(2k+2)$ could score both M's		
	$48 + 8k = 24k + 24$ $24 = 16k$		
	$k = \frac{3}{2}$		A1 oe
			[4]
			6 marks

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9. (a)	$\det \mathbf{M} = 3(-5) - (4)(2) = -15 - 8 = \underline{-23}$	$\underline{-23}$	B1
			[1]
(b)	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	Either, $3(2a-7) + 4(a-1) = 25$ or $2(2a-7) - 5(a-1) = -14$ or $\begin{pmatrix} 3(2a-7)+4(a-1) \\ 2(2a-7)-5(a-1) \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Any one correct equation (unsimplified) inside or outside matrices	A1
	giving $a = 5$	$a = 5$	A1
			[3]
(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4); = \underline{12}$ (units)^2	M1: $\frac{1}{2}(6)(\text{Their } a - 1)$ A1: 12 cao and cso	M1A1
	Note A(6, 0) is sometimes misinterpreted as (0, 6) – this is the wrong triangle and scores M0 e.g.1/2x6x3 = 9		
			[2]
(d)	$\text{Area}(OR'S') = \pm 23 \times (12)$	$\pm \det \mathbf{M} \times$ (their part (c) answer)	M1
		$\underline{276}$ (follow through provided area > 0)	A1 $\sqrt{}$
	A method not involving the determinant requires the coordinates of R′ to be calculated ((18, 12)) and then a <u>correct</u> method for the area e.g. (26x25 – 7x13 – 9x12 – 7x25) M1 = 276 A1		
			[2]
(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about (0, 0).	B1: Rotation, Rotates, Rotate, Rotating (not turn) B1: 90° anti-clockwise (or 270° clockwise) about (around/from etc.) (0, 0)	B1;B1
			[2]
(f)	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\mathbf{A}^{-1} = \frac{1}{(0)(0) - (1)(-1)}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	Applies $\mathbf{M}$(their $\mathbf{A}^{-1}$)	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$		A1
	NB some candidates state $\mathbf{M} = \mathbf{AB}$ and then calculate $\mathbf{MA}^{-1}$ or state $\mathbf{M} = \mathbf{BA}$ and then calculate $\mathbf{A}^{-1}\mathbf{M}$. These could score M0A0 M1A1ft and M1A1M0A0 respectively.		**[4]**
			14 marks
	Special case		
(f)	$\mathbf{M} = \mathbf{AB}$	$\mathbf{M} = \mathbf{AB}$, seen or implied.	M0
		$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A0
	$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & -4 \end{pmatrix}$	Applies (their $\mathbf{A}^{-1}$)$\mathbf{M}$	M1A1ft

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4(a)	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1 & 2 & 2\\1 & 1 & 4\end{pmatrix}=\begin{pmatrix}1 & 1 & 4\\1 & 2 & 2\end{pmatrix}$	Attempt to multiply the right way round with at least 4 correct elements	M1
	T' has coordinates (1,1), (1,2) and (4,2) or $\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}4\\2\end{pmatrix}$ **NOT just** $\begin{pmatrix}1 & 1 & 4\\1 & 2 & 2\end{pmatrix}$	Correct coordinates or vectors	A1
			(2)
(b)	**Reflection** in the line $y = x$	Reflection	B1
		$y = x$	B1
	Allow 'in the axis' 'about the line' $y = x$ etc. Provided both features are mentioned ignore any reference to the origin unless there is a clear contradiction.		
			(2)
(c)	$\mathbf{QR}=\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}=\begin{pmatrix}-2 & 0\\0 & 2\end{pmatrix}$	2 correct elements	M1
		Correct matrix	A1
	Note that $\mathbf{RQ}=\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}=\begin{pmatrix}10 & -4\\24 & -10\end{pmatrix}$ **scores M0A0 in (c) but allow all the marks in (d) and (e)**		
			(2)
(d)	$\det(\mathbf{QR}) = -2\times2-0=-4$	"-2"x"2" – "0"x"0"	M1
		-4	A1
	Answer only scores 2/2 $\dfrac{1}{\det(\mathbf{QR})}$ **scores M0**		**(2)**
(e)	Area of $T = \dfrac{1}{2}\times1\times3=\dfrac{3}{2}$	Correct area for T	B1
	Area of $T''=\dfrac{3}{2}\times4=6$	Attempt at $"\dfrac{3}{2}"\times\pm"4"$	M1
		6 or follow through their det(QR) x Their triangle area provided area > 0	A1ft

3. (a)	$\mathbf{A} = \begin{pmatrix} 1 & Ö2 \\ Ö2 & -1 \end{pmatrix}$			
(i)	$\mathbf{A}^2 = \begin{pmatrix} 1 & Ö2 \\ Ö2 & -1 \end{pmatrix}\begin{pmatrix} 1 & Ö2 \\ Ö2 & -1 \end{pmatrix}$			
	$= \begin{pmatrix} 1+2 & Ö2 - Ö2 \\ Ö2 - Ö2 & 2+1 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1	
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Correct answer	A1	
				(2)
(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	**Enlargement**; scale factor **3**, centre **(0, 0)**	B1; B1	
	Allow 'from' or 'about' for centre and 'O' or 'origin' for (0, 0)			
				(2)
(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$			
	Reflection; in the line $y = -x$.	**Reflection**; **y = −x**	B1; B1	
	Allow 'in the axis' 'about the line' $y = -x$ etc. **The question does not specify a <u>single</u> transformation so we would need to accept any combinations that are correct e.g. Anticlockwise rotation of 90° about the origin followed by a reflection in the x-axis is acceptable. In cases like these, the combination has to be <u>completely</u> correct and scored as B2 (no part marks). If in doubt consult your Team Leader.**			(2)
(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.			
	C is singular $\Rightarrow$ det C = 0. (Can be implied)	det C = 0	B1	
	Special Case $\dfrac{1}{9(k+1)-12k} = 0$ **B1(implied)M0A0**			
	$9(k+1) - 12k \ (= 0)$	Applies $9(k+1) - 12k$	M1	
	$9k + 9 = 12k$			
	$9 = 3k$			
	$k = 3$	$k = 3$	A1	
	k = 3 with no working can score full marks			(3)

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5.	$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.			
(a)	$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$			
	Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by both correct equations below.	M1	
	Do not allow this mark for other incorrect statements unless interpreted correctly later e.g. $\begin{pmatrix} 4 \\ 6 \end{pmatrix}\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$ would be M0 unless followed by correct equations or $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$			
	So, $-16 + 6a = 2$ and $4b - 12 = -8$ Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Any one correct equation. Any correct horizontal line	M1	
	giving $a = 3$ and $b = 1$.	Any one of $a = 3$ or $b = 1$.	A1	
		Both $a = 3$ and $b = 1$.	A1	
				(4)
(b)	$\det \mathbf{A} = 8 - (3)(1) = 5$	Finds determinant by applying $8 -$ their ab.	M1	
		$\det \mathbf{A} = 5$	A1	
	Special case: The equations -16 + 6b = 2 and 4a – 12 = -8 give a = 1 and b = 3. This comes from incorrect matrix multiplication. This will score nothing in (a) but allow all the marks in (b).			
	Note that $\det \mathbf{A} = \frac{1}{8-ab}$ scores M0 here but the following 2 marks are available. However, beware $\det \mathbf{A} = \frac{1}{8-ab} = \frac{1}{5} \Rightarrow area\ S = \frac{30}{\frac{1}{5}} = 150$ This scores M0A0 M1A0			
	Area S = (det **A**)(Area R)			
	Area $S = 5 \times 30 = 150$ (units)^2	$\frac{30}{\text{their } \det \mathbf{A}}$ or $30 \times (\text{their } \det \mathbf{A})$	M1	
		150 or ft answer	A1 $\sqrt{}$	
	If their det A < 0 then allow ft provided final answer > 0			(4)
	In (b) Candidates may take a more laborious route for the area scale factor and find the area of the unit square, for example, after the transformation represented by A. This needs to be a complete method to score any marks. Correctly establishing the area scale factor M1. Correct answer 5 A1. Then mark as original scheme.			