FP1 – Series Questions

1. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n+1)(2n-1), \text{ for all positive integers } n.$$

(5)

4. (a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right) = \frac{1}{4}n^2(n^2 + 2n + 13)$$

for all positive integers n.

(5)

(b) Hence find the exact value of

$$\sum_{r=16}^{30}\left(r^3 + 6r - 3\right)$$

(2)

7. (a) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3}n\left(an^2+b\right)$$

where a and b are integers to be found.

(4)

5\. (a) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, to prove that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers n.

(5)

(b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5)$$

(2)

3. (a) Show that

$$\sum_{r=1}^{n} r(r+2)(r+4) = \frac{1}{4}n(n+1)(n+4)(n+5).$$

(5)

(b) Hence evaluate $\sum_{r=21}^{30} r(r+2)(r+4)$.

(2)

1. (a) Show that $\sum_{r=1}^{n}(r^2 - r - 1) = \frac{1}{3}n(n^2 - 4)$.

(4)

(b) Hence, or otherwise, find the value of $\sum_{r=10}^{20}(r^2 - r - 1)$.

(2)