1 Show that the equation $x^3+2x^2-3x-2=0$ has a root between $x=1$ and $x=2$ **[2 marks]**

2 Given that $f(x)=x\sin x$, where x is in radians, show that $f(x)=0$ has a root in the interval $3<x<3.5$ **[2]**

3 **a** Show that the equation $x^3-4x-1=0$ has a root in the interval (2, 2.5) **[2]**

b Use the iterative formula $x_{n+1}=\sqrt{4+\frac{1}{x_n}}$, starting with $x_1=2$ to find x_2 and x_3 to 2 dp. **[3]**

4 **a** Use the iterative formula $x_{n+1}=\ln(5-x_n)$, starting with $x_1=1$ to find, to 2 decimal places, a root of the equation $e^x+x-5=0$ **[4]**

b Prove that your solution is correct to 2 decimal places. **[3]**

5 **a** Show that the equation $x^3-3x+1=0$ has a root between 1 and 2 **[2]**

b Taking 2 as a first approximation, use the Newton-Raphson process twice to find an approximation to the root of $x^3-3x+1=0$, to 2 dp. **[4]**

6 You are given that a particle's motion is modelled by $f(x)=2x^4-3x^3+4x$

a Use the Newton-Raphson process twice, taking $x=-1$ as the first approximation to find the negative root of the equation $f(x)=0$ to 2 decimal places. **[4]**

b Prove that your solution is correct to 2 dp. **[2]**

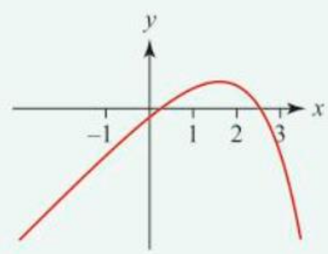
7 Use the trapezium rule with four strips to estimate the integral $\int_1^3 \sin^3 x\,dx$, to 3 sf. **[4]**

8 Use the trapezium rule with four ordinates to estimate the integral $\int_0^3 \tan^3 x\,dx$, to 3 sf. **[4]**

9 An object's temperature is modelled by $f(x)=5x-e^x$

a Prove that there is a root of $f(x)=0$ between $x=0$ and $x=0.5$ **[2]**

The graph of $y=f(x)$ is shown.



b Explain how you know from the graph that there is a root of $f(x)=0$ between 2 and 3 **[1]**

c Show that $x=2.5$ is the root, correct to 1 dp. **[3]**

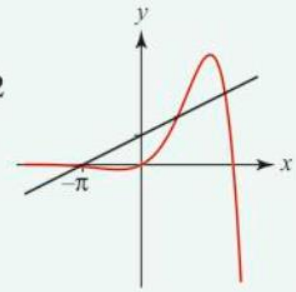
10 **a** Sketch, on the same axes, the graphs of $y=x+1$ and $y=\frac{4}{x}$ **[2]**

b Use your graphs to explain how many roots there are to the equation $x+1=\frac{4}{x}$ **[1]**

c Show that the equation $x+1=\frac{4}{x}$ has a root in the interval (1.5, 1.6) **[3]**

d Find the solutions to the equation $x+1=\frac{4}{x}$, give your answers to 3 significant figures. **[2]**

11 The graphs of $y = e^x \sin x$ and $y = x + 2$ are shown, where x is in radians.



a Explain how many solutions there are to the equation $e^x \sin x = x + 2$ **[1]**

b Show that one of the roots, α, is such that $1.2 < \alpha < 1.3$ **[3]**

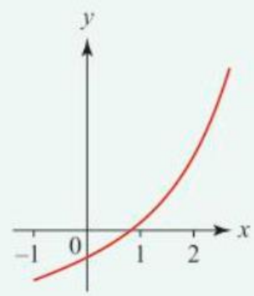
c Find an interval of size 0.1 that the other positive root lies within. **[3]**

12 a Show that the equation $x^3 - 3x^2 - 5 = 0$ can be written $x = \sqrt{\frac{5}{x} + 3x}$ **[2]**

b Use the iteration formula $x_{n+1} = \sqrt{\frac{5}{x_n} + 3x_n}$, starting with $x_1 = 3$ to find x_5

Give your answer to 2 decimal places. **[3]**

13 The graph of $y = 2e^x + 3x - 7$ is shown.



a Use the iteration formula $x_{n+1} = \frac{7 - 2e^{x_n}}{3}$ with $x_1 = 0.8$ to find x_2, x_3, x_4, x_5 to 2 decimal places. **[3]**

b Explain what is happening in this case. **[1]**

c Now derive a different iteration formula and, again using $x_1 = 0.8$. calculate x_2, x_3, x_4, x_5 to 2 dp. **[4]**

14 a Sketch the graphs of $y = x - 3$ and $y = \sqrt{x}$ on the same axes. **[2]**

b Use an appropriate iteration formula with $x_1 = 2$ to find a root of $\sqrt{x} = x - 3$ to 2 dp. **[4]**

c i Draw a suitable diagram to illustrate the results of the first two iterations. **[3]**

ii Write down the name of this diagram. **[1]**

15 a Sketch the graphs of $y = \ln x$ and $y = e^x - 5$ on the same axes. **[3]**

b Explain how many roots the equation $\ln x = e^x - 5$ has. **[1]**

c Show that one of the roots occurs between $x = 1.6$ and $x = 1.8$ **[3]**

d Use the Newton-Raphson process, to find this root correct to 2 decimal places. **[4]**

16 a Show that the equation $x^3 + 4x - 3 = 0$ can be written $x = a(b - x^3)$, where a and b are constants to be found. **[2]**

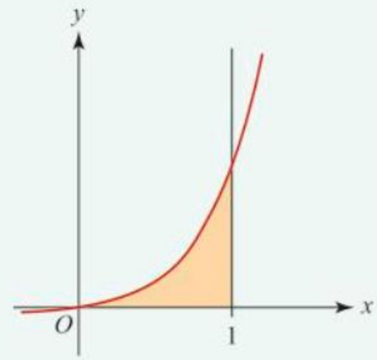
b Use the iteration formula $x_{n+1} = a(b - x_n^3)$ for the values of a and b found in part **a** with $x_1 = 0.1$ to find x_5 correct to 2 significant figures. **[3]**

c i Draw a suitable diagram to illustrate the results of the first 3 iterations. **[2]**

ii Write down the name of this diagram. **[1]**

17 Use the Newton-Raphson method to find, to 3 significant figures, the solution of the equation $x \sin x = 2 \ln x$, where x is in radians, which is near 2 **[5]**

18 Population growth can be modelled by the graph of $y = xe^{2x}$



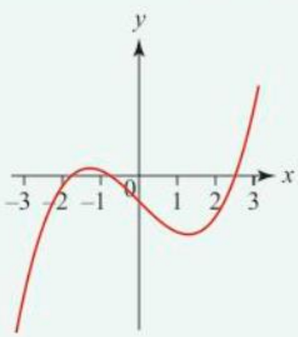
a Use the trapezium rule with five strips to estimate, to 4 significant figures, the area enclosed by the curve, the x-axis and the line $x = 1$ **[4]**

b State without further calculation whether this will be an overestimate or an underestimate of the actual area. Justify your answer. **[2]**

c **i** Use integration by parts to find the actual area. **[3]**

ii Calculate the percentage error in your approximation. **[2]**

19 The graph of $y = x^3 - 5x - 3$ is shown.



a How many solutions are there to the equation $x^3 - 5x - 3 = 0$? Justify your answer. **[1]**

b Show that $y = x^3 - 5x - 3 = 0$ can be written in the form $x = \pm\sqrt{a + \frac{b}{x}}$, where a and b are constants to be found. **[3]**

c Use the iteration formula $x_{n+1} = \sqrt{a + \frac{b}{x_n}}$ with the values you have found for a and b to calculate the positive root of the equation correct to 3 significant figures. **[3]**

d Use Newton-Raphson to find the largest negative root, correct to 2 significant figures. **[4]**

e Verify that the smallest negative root is -1.83 to 3 sf. **[2]**

20 Explain how the change of sign method will fail to find a root, α, to $f(x) = 0$ in these cases

a $f(x) = \frac{1}{x-3}$ for $2.5 < \alpha < 3.5$ **[2]**

b $f(x) = (3x-2)(2x-1)(x-4)$ for $0 < \alpha < 1$ **[2]**

21 Use the Newton-Raphson method to find a root, correct to 2 decimal places, to the equation $\sin^2 x = e^{-x}$, where x is in radians, using

a $x_1 = 1$ **[3]** **b** $x_1 = 3$ **[3]**

22 **a** Use the trapezium rule with five ordinates to estimate, to 3 significant figures, the area enclosed by the curve with equation $y = \sqrt{\ln x}$, the x-axis and the line $x = 2$ **[4]**

b Comment on the suggestion that the actual area is close to 0.5 **[2]**

23 $f(x) = x\ln x - 1, x > 0$

a Find an interval of size 0.2 that contains the solution to $f(x) = 0$ **[3]**

b Use Newton-Raphson to approximate the root of the equation $f(x) = 0$

Ensure your answer is correct to 3 decimal places. **[8]**