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## IYGB - FMI PAPER 0 - QUESTION 1

- BY CONSERVATION OF MOMENTUM

$$\Rightarrow m \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \lambda m \begin{pmatrix} -3 \\ 3 \end{pmatrix} = (m + \lambda m) \begin{pmatrix} k \\ k \end{pmatrix}$$

- DIVIDING BY M

$$\Rightarrow \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \end{pmatrix} = (1 + \lambda) \begin{pmatrix} k \\ k \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 6 - 3\lambda \\ -2 + 3\lambda \end{pmatrix} = \begin{pmatrix} k(1 + \lambda) \\ k(1 + \lambda) \end{pmatrix}$$

- Thus we have

$$\left. \begin{array}{l} 6 - 3\lambda = k(1 + \lambda) \\ 3\lambda - 2 = k(1 + \lambda) \end{array} \right\} \Rightarrow 6 - 3\lambda = 3\lambda - 2$$

$$8 = 6\lambda$$

$$\lambda = \frac{4}{3}$$

- Finally we have

$$\Rightarrow 6 - 3\lambda = k(1 + \lambda)$$

$$\Rightarrow 6 - 3\left(\frac{4}{3}\right) = k\left(\frac{4}{3} + 1\right)$$

$$\Rightarrow 6 - 4 = \frac{7}{3}k$$

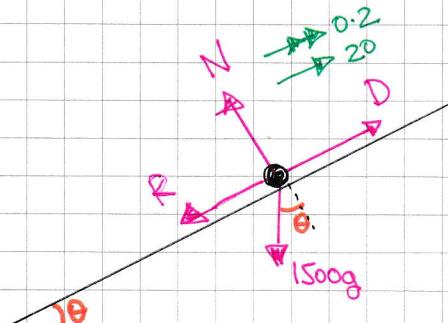
$$\Rightarrow 2 = \frac{7}{3}k$$

$$\Rightarrow k = \frac{6}{7}$$

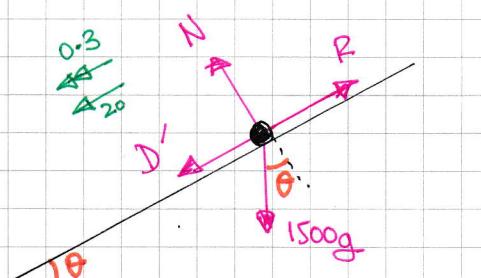
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## IYGB-FMI PAPER 0 - QUESTION 2

DRAW SEPARATE DIAGRAM FOR THE UPHILL & DOWNTILL MOTION



$$P = 96 \text{ kW}$$



$$P' = 60 \text{ kW}$$

FIRSTLY CALCULATE THE TRACTIVE (DRIVING) FORCE IN EACH CASE

$$\Rightarrow P = Dv$$

$$\Rightarrow 96000 = D \times 20$$

$$\Rightarrow D = \underline{\underline{4800}}$$

$$\Rightarrow P' = D'v'$$

$$\Rightarrow 60000 = D' \times 20$$

$$\Rightarrow D' = \underline{\underline{3000}}$$

NEXT WRITE THE EQUATION OF MOTION IN EACH CASE

$$\Rightarrow D - R - 1500g \sin \theta = mu$$

$$\Rightarrow 4800 - R - 1500g \sin \theta = 1500 \times 0.2$$

$$\Rightarrow 4500 = R + 1500g \sin \theta$$

$$\Rightarrow D' + 1500g \sin \theta - R = ma'$$

$$\Rightarrow 3000 + 1500g \sin \theta - R = 1500 \times 0.3$$

$$\Rightarrow 2550 = R - 1500g \sin \theta$$

ADDING THE EQUATIONS YIELDS

$$7050 = 2R$$

$$R = \underline{\underline{3525 \text{ N}}}$$

SUBTRACTING THE EQUATIONS GIVES

$$1950 = 3000g \sin \theta$$

$$\sin \theta = \frac{13}{1950}$$

$$\theta \approx \underline{\underline{3.80^\circ}}$$

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## IYGB - FM11 PAPER 0 - QUESTION 3

START BY OBTAINING THE COEFFICIENT OF FRICTION - OR THE CONSTANT  
FRICTIONAL FORCE



TAKING THE LEVEL OF "CD" AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$$\Rightarrow \cancel{KE_A} + \cancel{PE_A} + \cancel{W_{in}} - W_{out} = \cancel{KE_B} + \cancel{PE_B}$$

$$\Rightarrow mg(2x) - F \times d = 0$$

$$Fd = 2mgx$$

NOW BY ENERGY TRANSFER FROM A TO THE MIDPOINT OF CD

$$\Rightarrow \cancel{KE_A} + \cancel{PE_A} + \cancel{W_{in}} - W_{out} = KE_M + \cancel{PE_M}$$

$$\Rightarrow mg(2x) - F \times \frac{1}{2}d = \frac{1}{2}mv^2$$

$$\Rightarrow 4mgx - Fd = mv^2$$

$$\Rightarrow 4mgx - 2mgx = mv^2$$

$$\Rightarrow 2mgx = mv^2$$

$$\Rightarrow v^2 = 2gx$$

$$\Rightarrow h = \sqrt{2gx}$$

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## IYGB - FMI PAPER O - QUESTION 4

a) using  $\underline{I} = m\underline{v} - m\underline{u}$

$$\Rightarrow -8\underline{i} + 4\underline{j} = 0.25(12\underline{i} + 20\underline{j}) - 0.25\underline{u}$$

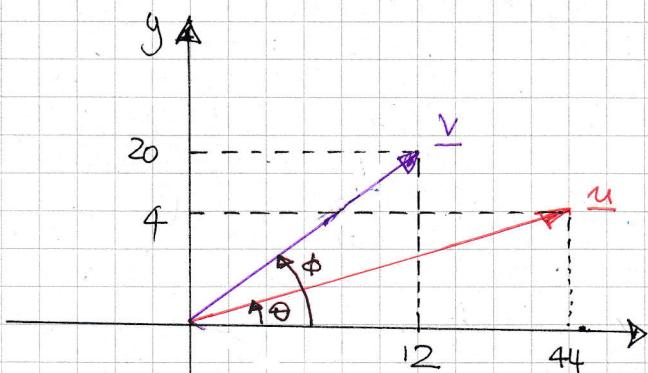
$$\Rightarrow -32\underline{i} + 16\underline{j} = 12\underline{i} + 20\underline{j} - \underline{u}$$

$$\Rightarrow \underline{u} = 44\underline{i} + 4\underline{j}$$

$$\Rightarrow |\underline{u}| = \sqrt{44^2 + 4^2} = \sqrt{1952}$$

$$\Rightarrow |\underline{u}| \approx 44.2 \text{ ms}^{-1}$$

b) looking at the diagram below



$$\text{deflection angle} = \phi - \theta$$

$$= \arctan\left(\frac{20}{12}\right) - \arctan\left(\frac{4}{44}\right)$$

$$= \arctan\left(\frac{5}{3}\right) - \arctan\left(\frac{1}{11}\right)$$

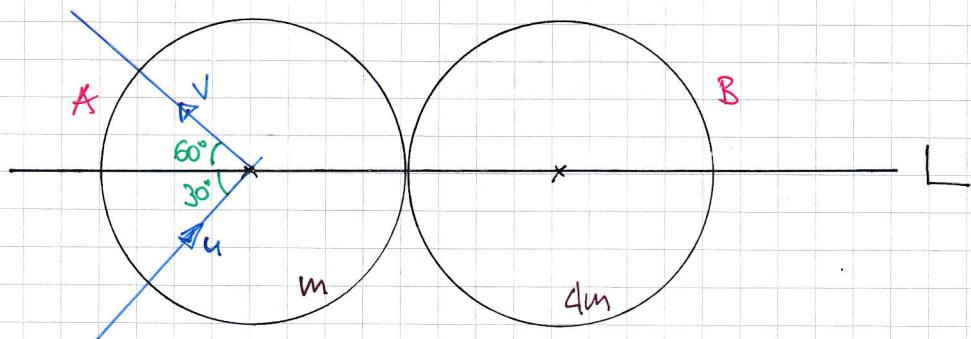
$$= 59.036^\circ - 5.1944^\circ$$

$$\approx 53.84^\circ$$

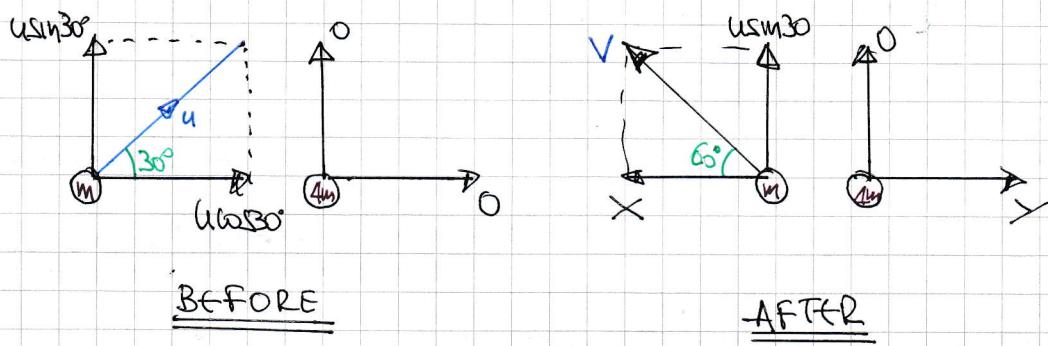
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## IYGB - FMI PAPER 0 - QUESTION 5

DRAWN & STANDARD OBLIQUE COLLISION DIAGRAM



DRAW BEFORE & AFTER & NOTE THAT THERE IS NO MOMENTUM EXCHANGE IN A DIRECTION PERPENDICULAR TO "L"



BY CONSERVATION OF MOMENTUM ALONG "L"

$$mu \cos 30^\circ + 0 = -mx + 4my$$

$$-x + 4y = \frac{\sqrt{3}}{2}u \quad \text{--- (I)}$$

BY RESTITUTION ALONG "L"

$$e = \frac{S_{AP}}{S_{PP}} \implies e = \frac{x+y}{u \cos 30^\circ}$$

$$\implies x+y = eu \cos 30^\circ$$

$$\implies x+y = \frac{\sqrt{3}}{2}eu \quad \text{--- (II)}$$

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## IYGB - FMI PAPER 0 - QUESTIONS

AS THERE IS NO MOMENTUM EXCHANGE PERPENDICULAR TO "L"

$$\begin{aligned} X &= V \cos 60 \\ \boxed{X &= \frac{1}{2}V} \end{aligned}$$

BY GEOMETRY

$$V \sin 60 = u \sin 30$$

$$\frac{\sqrt{3}}{2}V = \frac{1}{2}u$$

$$\boxed{u = \sqrt{3}V} \quad \text{--- (IV)}$$

SOLVING (I) & (II) FOR X & Y BY ADDING

$$\Rightarrow SY = \frac{\sqrt{3}}{2}u + \frac{\sqrt{3}}{2}eu$$

$$\Rightarrow Y = \frac{\sqrt{3}}{10}u + \frac{\sqrt{3}}{10}eu$$

$$\Rightarrow X = \frac{\sqrt{3}}{2}eu - Y$$

$$\Rightarrow X = \frac{\sqrt{3}}{2}eu - \frac{\sqrt{3}}{10}u - \frac{\sqrt{3}}{10}eu$$

$$\Rightarrow \boxed{X = \frac{2}{5}\sqrt{3}eu - \frac{\sqrt{3}}{10}u}$$

USING THIS X WITH (III) & (IV)

$$\Rightarrow X = \frac{1}{2}V$$

$$\Rightarrow 2X = V$$

$$\Rightarrow 2\sqrt{3}X = \sqrt{3}V$$

$$\Rightarrow 2\sqrt{3}X = u$$

$$\Rightarrow 2\sqrt{3} \left[ \frac{2}{5}\sqrt{3}eu - \frac{\sqrt{3}}{10}u \right] = u$$

$$\Rightarrow 2\sqrt{3} \left( \frac{2}{5}\sqrt{3}e - \frac{\sqrt{3}}{10} \right) = 1$$

$$\Rightarrow \frac{12}{5}e - \frac{3}{5} = 1$$

$$\Rightarrow 12e - 3 = 5$$

$$\Rightarrow 12e = 8$$

$$\therefore e = \frac{2}{3}$$

## IYGB - FMI PAPER 0 - QUESTION 6

a)

BY CONSIDERING ENERGIES TAKING THE LEVEL OF "P" AS THE ZERO  
GRAVITATIONAL POTENTIAL WE OBTAIN

$$\Rightarrow KE_p + PE_p + EE_p = KE_A + PE_A + EE_A$$

(IGNORED  $W_{in}$  &  $W_{out}$  & Friction)

$$\Rightarrow 0 = \frac{1}{2}mv^2 - mgx + \frac{\lambda}{2l}(x-l)^2$$

$$\Rightarrow 2mgx - \frac{\lambda}{l}(x-l)^2 = mv^2$$

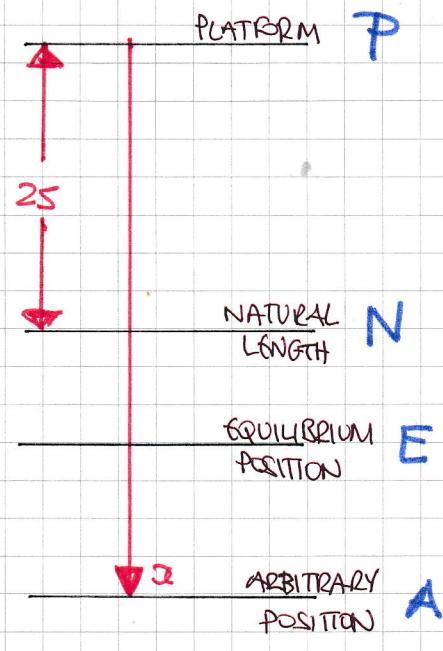
$$\Rightarrow 1470x - 147(x-25)^2 = 75v^2$$

$$\Rightarrow 1470x - 147(x^2 - 50x + 625) = 75v^2$$

$$\Rightarrow 1470x - 147x^2 + 7350x - 91875 = 75v^2$$

$$\Rightarrow 75v^2 = -147x^2 + 8820x - 91875$$

$$\Rightarrow 25v^2 = -49x^2 + 2940x - 30625$$



$\downarrow \div 3$   
 $\swarrow$  AS REQUIRED

$m = 75$   
 $\lambda = 3675$   
 $l = 25$

NOW MAXIMUM VALUE OF  $x$  WILL OCCUR WITH  $v=0$

$$\Rightarrow 0 = -49x^2 + 2940x - 30625$$

$$\Rightarrow 49x^2 - 2940x + 30625 = 0 \quad \Rightarrow \div 49$$

$$\Rightarrow x^2 - 60x + 625 = 0$$

$$\Rightarrow (x-30)^2 - 900 + 625 = 0$$

$$\Rightarrow (x-30)^2 = 275$$

$$\Rightarrow x-30 = \begin{cases} 5\sqrt{11} \\ -5\sqrt{11} \end{cases}$$

$$\Rightarrow x = \begin{cases} 30 + 5\sqrt{11} \approx 46.58m \\ 30 - 5\sqrt{11} \approx 13.42 \end{cases} \text{ (STRING WILL SLACK)}$$

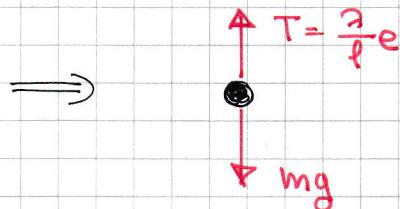
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## IYGB - FULL PAPER 0 - QUESTION 6

b)

NOW FOR MAX SPEED  $\Rightarrow$  ZERO ACCELERATION

$\Rightarrow$  EQUILIBRIUM



$$\Rightarrow \frac{3}{4}e = mg$$

$$\Rightarrow e = \frac{4mg}{3}$$

$$\Rightarrow e = \frac{25 \times 75 \times 9.8}{3675}$$

$$\Rightarrow e = 5$$

$$\Rightarrow x = 25 + 5 = 30$$

USING THE ENERGY EQUATION WITH  $x=30$

$$\Rightarrow 25V^2 = -49x^2 + 2940x - 30625$$

$$\Rightarrow 25V^2 = -49 \times 30^2 + 2940 \times 30 - 30625$$

$$\Rightarrow 25V^2 = 13475$$

$$\Rightarrow V^2 = 539$$

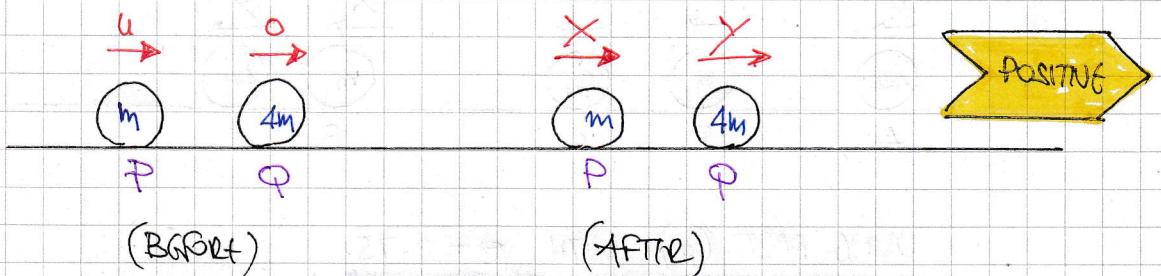
$$\Rightarrow |V| \approx 23.22 \text{ ms}^{-1}$$

$(7\sqrt{11})$

— + —

## IYGB - FMI PAPER 0 - QUESTION 7

u



## BY CONSERVATION OF MOMENTUM

$$\Rightarrow m u + o = m X + 4m Y$$

$$\Rightarrow X + 4Y = u$$

BY RESTITUTION

$$e = \frac{SGP}{APP}$$

$$e = \frac{Y-X}{S}$$

$$-x + y = e^y$$

## ADDING THE EQUATIONS GIVES

$$5Y = u + eu$$

$$5Y = u(1+e)$$

$$Y = \frac{1}{5}u(e+1) //$$

$$\text{AND } Y = X + \epsilon u$$

$$\frac{1}{5}eu + \frac{1}{5}u = X + eu$$

$$X = \frac{1}{5}u - \frac{4}{5}\epsilon u$$

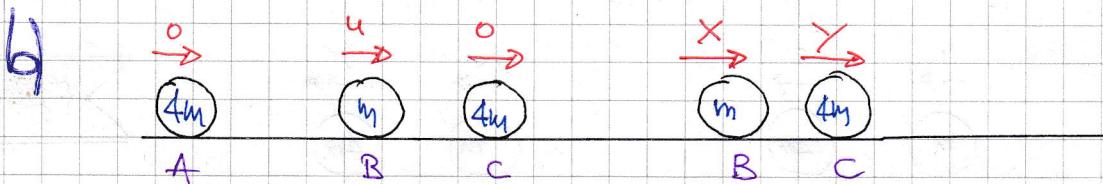
$$X = \frac{1}{5} u (1 - 4e)$$

$$\therefore \text{SPEED IS } |x| = \frac{1}{5}u(4e^{-1})$$

## "BACKWARDS"!!

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# IYGB - FULL PAPER 0 - QUESTION 7



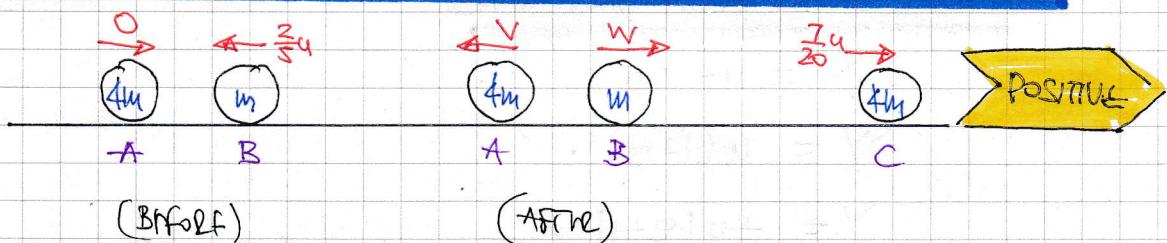
USING PART (a) WITH  $\epsilon = 0.75$

$$X = \frac{1}{5}u(1 - 4(0.75)) = -\frac{2}{5}u \quad (\text{Backwards})$$

$$Y = \frac{1}{5}u(1 + 0.75) = \frac{7}{20}u$$

∴ AS B. REBOUNDS ANOTHER  
COLLISION BTW FN & B

NOW THE COLLISION BETWEEN A & B AND ITS AFTERMATH



BY CONSERVATION OF MOMENTUM

$$0 - \frac{2}{5}mv = -4mV + mw$$

$$W - 4V = -\frac{2}{5}u$$

By RESTITUTION

$$e = \frac{S6P}{ATP}$$

$$0.75 = \frac{V+W}{2U}$$

$$V+W = \frac{3}{10} u$$

$$4V + 4W = \frac{6}{5}U$$

## ADDING THE EQUATIONS

$$5w = \frac{4}{3}u$$

$$W = \frac{4}{25}u$$

(TO THE "RIGHT")

AS  $\frac{4}{25}w < \frac{7}{20}4$  NO MORE  
COLLISIONS BREAKDOWN B & C