Bronze

1. Given that $z = 2 - 2\text{i}$ and $w = -\sqrt{3} + \text{i}$,

(a) find the modulus and argument of wz^2.

(6)

(b) Show on an Argand diagram the points A, B and C which represent z, w and wz^2 respectively, and determine the size of angle BOC.

(4)
(Total 10 marks)

Silver

2. Given that $z = 4\left(\cos\frac{3\pi}{4} + \text{i}\sin\frac{3\pi}{4}\right)$ and $w = 1 - \text{i}\sqrt{3}$, find

(a) $\left|\frac{z}{w}\right|$,

(3)

(b) arg $\left(\frac{z}{w}\right)$, in radians as a multiple of π.

(3)

(c) On an Argand diagram, plot points A, B, C and D representing the complex numbers z, w, $\left(\frac{z}{w}\right)$ and 4, respectively.

(3)

(d) Show that $\angle AOC = \angle DOB$.

(3)

(e) Find the area of triangle AOC.

(2)
(Total 14 marks)

Gold

3. The point P represents a complex number z on an Argand diagram such that

$$|z - 3| = 2\,|z|.$$

(a) Show that, as z varies, the locus of P is a circle, and give the coordinates of the centre and the radius of the circle.

(5)

The point Q represents a complex number z on an Argand diagram such that

$$|z + 3| = |z - \mathrm{i}\sqrt{3}|.$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(5)

(c) On your diagram shade the region which satisfies

$$|z - 3| \geq 2\,|z| \text{ and } |z + 3| \geq |z - \mathrm{i}\sqrt{3}|.$$

(2)

(Total 12 marks)