YEAR 2 PURE

BINOMIAL EXPANSION

Chapter 4

Binomial series

option A

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

option B

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \quad (|x|<1,\ n \in \mathbb{R})$$

Given in the formula booklet

Which formula to use?

$(a+b)^n$ positive integers $n \in \mathbb{N}$

$(1+x)^n$ positive, negatives + fractions

Possible Questions

$(a+b)^n$	$(1+x)^n$
$(2+x)^5$	$(1+x)^{1/2}$
$(2+3x)^4$	$(1+3x)^{-2}$
$(1-2x)^4$	$(1+2x)^{-1/3}$
	$(2+4x)^{-5}$

$(1+x)^n$ Terms

1st term		2nd term		3rd term		4th term
1	$+$	nx	$+$	$\dfrac{n(n-1)}{2\times 1}x^2$	$+$	$\dfrac{n(n-1)(n-2)}{3\times 2\times 1}x^3$

5th term

$$+\ \frac{n(n-1)(n-2)(n-3)}{4\times 3\times 2\times 1}x^4 \ + \text{etc} \ldots\ldots$$

Identify n and x

$$\sqrt{1-3x} = (1-3x)^{1/2}$$ $n = 1/2$, $x = -3x$

$$\frac{1}{(1+2x)^2} = (1+2x)^{-2}$$ $n = -2$, $x = 2x$

$$(2+4x)^{-2} = 2^{-2}(1+2x)^{-2}$$
$$= \frac{1}{4}(1+2x)^{-2}$$ $n = -2$, $x = 2x$

Dont forget to multiply all terms by $\frac{1}{4}$ at the end.

Combining Expansions

$$\frac{(1+x)^{\frac{1}{2}}}{(1-x)^{-\frac{1}{2}}} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$$

$$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2\right)$$

Expand (1st 3 terms) $= 1 + x + \frac{1}{2}x^2$