KS5 "Full Coverage": Trigonometric Equations and Expressions (Yr2)

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Double angle formula for sin

[Edexcel C3 June 2009 Q8a] Write down $sin\, 2x$ in terms of $sin\, x$ and $cos\, x$.

$sin\, 2x = \ldots\ldots\ldots\ldots\ldots\ldots$

Question 2

Categorisation: Simplify an expression involving a double angle formula.

[Edexcel C3 June 2016 Q8a Edited] Write $2\, cot\, 2x + \, tan\, x$, $x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$

as a single trigonometric ratio.

$2\, cot\, 2x + \, tan\, x \equiv \ldots\ldots\ldots\ldots\ldots\ldots$

Question 3

Categorisation: As above.

[Edexcel A2 Specimen Papers P2 Q13a Edited]

Show that

$cosec\ 2x + \ cot\ 2x \equiv cot\ x\ ,\qquad x \neq 90n^{\circ}, n \in \mathbb{Z}$

.....

Question 4

Categorisation: As above.

[Edexcel C3 June 2017 Q9a Edited]

Prove that

$sin\ 2x - \ tan\ x \equiv \ tan\ x\ cos\ 2x \qquad x \neq (2n+1)90^{\circ}\ ,\quad n \in \mathbb{Z}$

.....

Question 5

Categorisation: Recognise that the double angle formula for $\boldsymbol{cos\ 2x \equiv cos^2 x - sin^2 x}$ is the difference of two squares, allowing for possible further simplification.

[Edexcel C3 June 2015 Q8a] Prove that

$$sec\ 2A + \ tan\ 2A \equiv \frac{cos\ A + sin\ A}{cos\ A - \ sin\ A}$$

.....

Question 6

Categorisation: Solve a trigonometric equation using that identities $\boldsymbol{1 + tan^2 x \equiv sec^2 x}$ and $\boldsymbol{1 + cot^2 x \equiv cosec^2 x}$

[Edexcel C3 Jan 2012 Q5]

Solve, for $0 \leq \theta \leq 180^\circ$,

$$2\ cot^{\ 2} 3\theta = 7\ cosec\ 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

.....

Question 7

Categorisation: Use the Addition Formulae an simplify an expression.

[Edexcel C3 June 2015 Q1c]

Given that $tan\ \theta^\circ = p$, where p is a constant, $p \neq \pm 1$, use standard trigonometric identities, to find $cot\ (\theta - 45)^\circ$ in terms of p .

$cot\ (\theta - 45)^\circ =$ .....

Question 8

Categorisation: Use the Addition Formulae to find sin/cos/tan of 15, 75, etc.

[Edexcel C3 Jan 2009 Q6b Edited] Using $sin\ (\theta - \alpha) = sin\ \theta\ cos\ \alpha - cos\ \theta\ sin\ \alpha$, or otherwise, show that

$$sin\ 15^\circ = \frac{1}{4}\left(\sqrt{x} - \sqrt{y}\right)$$

where x and y are integers to be found.

.....

Question 9

Categorisation: Recognise that $\boldsymbol{sin(x) \equiv cos(90 - x)}$

[Edexcel C3 June 2013 Q3a Edited]

Given that $2\ cos\ (x + 50)^\circ = sin\ (x + 40)^\circ$, show that $tan\ x^\circ = k\ tan\ 40^\circ$, where k is a constant to be found.

.....

Question 10

Categorisation: Use the double-angle formulae backwards.

[Edexcel C3 Jan 2013 Q6i]

Without using a calculator, find the exact value of

$$\left(sin\ 22.5^\circ + cos\ 22.5^\circ \right)^2$$

You must show each stage of your working.

.....

Question 11

Categorisation: Use the Additional Formulae for tan.

[Edexcel C3 Jan 2012 Q8b Edited] Given that

$$tan\ (A + B) = \frac{tan\ A + tan\ B}{1 - tan\ A\ tan\ B}$$

Deduce that

$$tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + b\ tan\ \theta}{c + d\ tan\ \theta}$$

where b , c and d are constants to be found.

.....

Question 12

Categorisation: Solve an equation given a previously proven identity.

[Edexcel C3 June 2006 Q6c Edited]

It can be shown that $csc^4\theta - cot^4\theta = csc^2\theta + cot^2\theta$

Solve, for $90^\circ < \theta < 180^\circ$,

$$csc^4\theta - cot^4\theta = 2 - cot\,\theta$$

$\theta =$ ..... $^\circ$

Question 13

Categorisation: As above.

[Edexcel C3 Jan 2012 Q8c Edited]

$$tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\,tan\,\theta}{\sqrt{3} - tan\,\theta}$$

Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3}\,tan\,\theta = \left(\sqrt{3} - tan\,\theta\right) tan\,(\pi - \theta)$$

Give your answers as multiples of π.

.....

Question 14

Categorisation: Triple angle formula for sin and cos.

[Edexcel C3 Jan 2009 Q6ai Edited]

By writing $3\theta = (2\theta + \theta)$, show that

$$sin\ 3\theta = a\ sin\ \theta - b\ sin^3\theta$$

where a and b are constants to be found.

.....

Question 15

Categorisation: As above.

[Edexcel C3 Jan 2008 Q6a]

Use the double angle formulae and the identity

$$cos\ (A + B) \equiv\ cos\ A\ cos\ B -\ sin\ A\ sin\ B$$

to obtain an expression for $cos\ 3x$ in terms of powers of $cos\ x$ only.

$cos\ 3x \equiv$ .....

Question 16

Categorisation: Use double angle formulae to convert parametric equations to a Cartesian one.

[Edexcel C4 June 2013 Q4b Edited] A curve *C* has parametric equations

$x = 2\,sin\,t\,,\quad y = 1 - \,cos\,2t\,,\quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

Find a cartesian equation for *C* in the form

$y = f(x)\,,\quad -2 \le x \le 2$

$y =$ .....

Question 17

Categorisation: Use small-angle approximations.

[Edexcel A2 Specimen Papers P2 Q2a Edited] Given that θ is small, use the small angle approximation for $cos\,\theta$ to show that

$$1 + 4\,cos\,\theta + 3\,cos^{\,2}\theta \approx a + b\theta^2$$

where a and b are constants to be found.

.....

Question 18

Categorisation: As above

When θ is small and measured in radians, find the approximate value of

$$\frac{\theta(\sin\theta + \tan\theta)}{\cos\theta - 1}$$

.....

Question 19

Categorisation: As above.

[Edexcel A2 Specimen Papers P2 Q2bi Edited]

Given that θ is small, using the small angle approximation for $\cos\theta$ it can be shown that

$$1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$$

Adele uses $\theta = 5^\circ$ to test this approximation.
Adele's working is shown below.

Using my calculator, $1 + 4\cos(5°) + 3\cos^2(5°) = 7.962$, to 3 decimal places.
Using the approximation $8 - 5\theta^2$ gives $8 - 5(5)^2 = -117$
Therefore, $1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ is not true for $\theta = 5°$

Identify the mistake made by Adele in her working.

.....

Question 20

Categorisation: Write $a\ sin\ x + b\ cos\ x$ in the form $R\ sin(x + \alpha)$ or $R\ cos\ (x + \alpha)$

[Edexcel C3 June 2015 Q3a]

$$g(\theta) = 4\ cos\ 2\theta + 2\ sin\ 2\theta$$

Given that $(\theta) = R\ cos\ (2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^{\circ}$, find the exact value of R and the value of α to 2 decimal places.

.....

Question 21

Categorisation: As above.

[Edexcel A2 Specimen Papers P1 Q13a]

Express $2\ sin\ \theta - 1.5\ cos\ \theta$ in the form $R\ sin\ (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 decimal places.

.....

Question 22

Categorisation: As above, but used for modelling.

[Edexcel A2 Specimen Papers P1 Q13b Edited]

It can be shown that $2\ sin\ \theta - 1.5\ cos\ \theta = 2.5\ sin\ (\theta - 0.6435)$ where θ is measured in radians.

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2\ sin\left(\frac{4\pi t}{25}\right) - 1.5\ cos\left(\frac{4\pi t}{25}\right), 0 \leq t \leq 24$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to find the depth of water at 00:00 hours on 18th October 2017.

..... m

Question 23

Categorisation: Determine the maximum or minimum value of a trigonometric expression.

[Edexcel A2 Specimen Papers P1 Q13c Edited] (Continued from above) Use Tom's model to find the maximum depth of water.

..... m

Question 24

Categorisation: Determine the time/value of x/t for which the maximum/minimum occurs.

[Edexcel A2 Specimen Papers P1 Q13d Edited] (Continued from above)

Use Tom's model to find the time, in the afternoon, when the maximum depth of water occurs.
Give your answer to the nearest minute.

..... : .....

Question 25

Categorisation: Describe how a model would be adapted.

[Edexcel A2 SAM P2 Q13d Edited] It can be shown that $10\ cos\ \theta - 3\ sin\ \theta = \sqrt{109}\ cos\ (\theta + 16.70)$

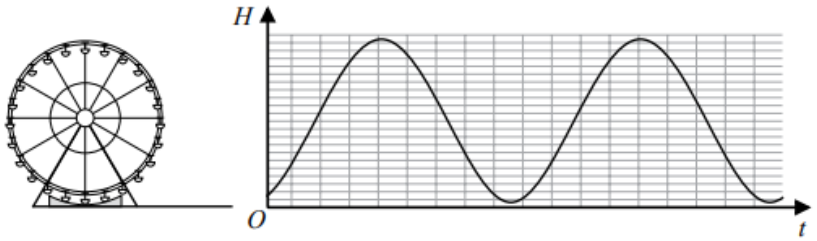

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = 11 - 10\ cos\ (80t)^\circ + 3\ sin\ (80t)^\circ$$

Figure 3 shows the graph of H against t for two complete cycles of the wheel. It is decided that, to increase profits, the speed of the wheel is to be increased. How would you adapt the equation of the model to reflect this increase in speed?

Question 26

Categorisation: Determine the time taken for a trigonometric function to make one or more 'cycles'.

[Edexcel A2 SAM P2 Q13c Edited] (Continued from above) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

..... minutes

and ..... seconds

Question 27

Categorisation: Determine the maximum or minimum value of an trigonometric function used within another expression, e.g. a division.

[Edexcel C3 Jan 2013 Q4bi Edited]

$6\cos\theta + 8\sin\theta$ can be expressed as $10\cos(\theta - 0.927)$.

$$p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, 0 < \theta < 2\pi$$

Calculate the maximum value of $p(\theta)$.

Maximum = ...

Question 28

Categorisation: Consider the number of solutions of a trigonometric equation.

[Edexcel C3 June 2015 Q3c]

$$g(\theta) = 4\cos 2\theta + 2\sin 2\theta$$

It can be shown that $g(\theta) = \sqrt{20}\cos\left(2\theta - 26.57^{\circ}\right)$

Given that k is a constant and the equation $g(\theta) = k$ has no solutions, state the range of possible values of k .

.....

Question 29

Categorisation: Solve an equation involving arcsin or arccos.

[Edexcel C3 June 2016 Q7b]

$g(x) = \arcsin x\,, \quad -1 \leq x \leq 1$

Find the exact value of x for which

$$3g(x+1) + \pi = 0$$

.....

Question 30

Categorisation: Put an expression $a\ sin\ t + b\ cos\ t$ in the form $R\ sin\ (t + \alpha)$ in a context where it is not obvious that you might need to do so.

[Edexcel C4 June 2014(R) Q8c Edited]

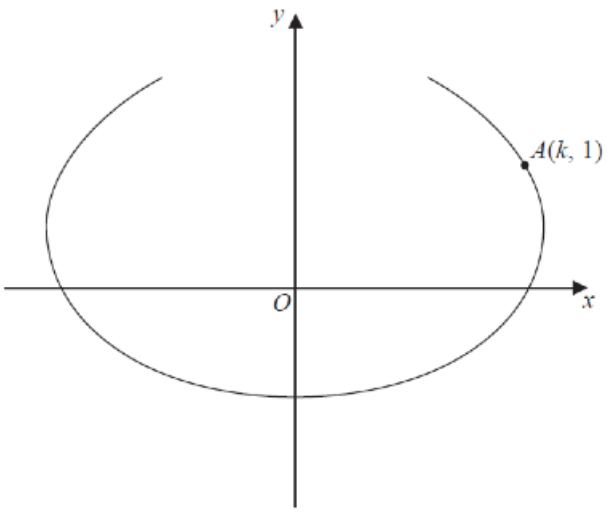

Figure 3

The curve shown in Figure 3 has parametric equations

$x = t - 4\ sin\ t\ ,\ \ y = 1 - 2\ cos\ t\ ,\ \ -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$

The point A , with coordinates $(k, 1)$, lies on the curve.

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$.

Given that $k = 4 - \frac{\pi}{2}$, and that $\frac{dy}{dx} = \frac{2\ sin\ t}{1-4\ cos\ t}$, find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

$t =$ .....

Answers

Question 1

$sin\ 2x = 2\ sin\ x\ cos\ x$

Question 2

$2\ cot\ 2x + \ tan\ x \equiv \ cot\ x$

Question 3

$$
\begin{aligned}
&cosec\ 2x + \cot 2x \\
&\equiv \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} \\
&\equiv \frac{1 + 2\cos^2 x - 1}{2\sin x \cos x} \equiv \frac{2\cos^2 x}{2\sin x \cos x} \\
&\equiv \frac{\cos x}{\sin x} \equiv \cot x
\end{aligned}
$$

Question 4

$$
\begin{aligned}
&\sin 2x - \tan x \equiv \tan x \cos 2x \\
&\text{LHS} = 2\sin x \cos x - \frac{\sin x}{\cos x} \\
&= \frac{2\sin x \cos^2 x - \sin x}{\cos x} \\
&= \left(\frac{\sin x}{\cos x}\right)(2\cos^2 x - 1) \\
&= \tan x \cos 2x
\end{aligned}
$$

Question 5

$$
\begin{aligned}
&\sec 2A + \tan 2A \equiv \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A} \\
&\equiv \frac{1 + 2\sin A\ cosA}{\cos^2 A - \sin^2 A} \\
&\equiv \frac{\sin^2 A + \cos^2 A + 2\sin A\, cosA}{(\cos A + \sin A)(\cos A - \sin A)} \\
&\equiv \frac{(\cos A + \sin A)^2}{(\cos A + \sin A)(\cos A - \sin A)} \\
&\equiv \frac{\cos A + \sin A}{\cos A - \sin A}
\end{aligned}
$$

(note that it is easier to work from both the LHS and RHS and meet in the middle)

Question 6

$\theta = 6.5$ or $\theta = 53.5$ or $\theta = 126.5$ or $\theta = 173.5$

Question 7

$cot\ (\theta - 45)^\circ = \frac{1+p}{p-1}$

Question 8

$x = 6$, $y = 2$

Question 9

$$k = \frac{1}{3}$$

Question 10

$1 + \frac{\sqrt{2}}{2}$

Question 11

$b = \sqrt{3}$, $c = \sqrt{3}$, $d = -1$

Question 12

$\theta =$ 135 $^\circ$

Question 13

$\theta = \frac{5\pi}{12}$ or $\theta = \frac{11\pi}{12}$

Question 14

$a = 3$, $b = 4$

Question 15

$\cos 3x$
$\equiv 4\ cos\ ^3x - 3\ cos\ x$

Question 16

$y = \frac{x^2}{2}$

Question 17

$a = 8$, $b = -5$

Question 18

-4

Question 19

"degree OR degrees OR radian OR radians"

Question 20

$R = \sqrt{20}$, $\alpha = 26.57$

Question 21

$R = 2.5$, $\alpha = 0.6435$

Question 22

4.5 m

Question 23

8.5 m

Question 24

$4{:}\ 54$

Question 25

Increase the 80 in the formula

Question 26

6 minutes and and 32 seconds

Question 27

Maximum = 2

Question 28

$k > \sqrt{20}$ or $k < -\sqrt{20}$

Question 29

$$x = -1 - \frac{\sqrt{3}}{2}$$

Question 30

$t =$ 0.6077