

1)

$$y = 2x^3 + 5x^2 - 7x + 10$$

$$\int y \, dx = \frac{2x^4}{4} + \frac{5x^3}{3} - \frac{7x^2}{2} + 10x + c$$

$$= \frac{1}{2}x^4 + \frac{5}{3}x^3 - \frac{7}{2}x^2 + 10x + c$$

2)

$$\int 3x^2 + 7x - 2 \, dx$$

$$= \frac{3x^3}{3} + \frac{7x^2}{2} - 2x + c$$

$$= x^3 + \frac{7}{2}x^2 - 2x + c$$

3)

$$\int (x+4)(x-3) \, dx$$

$$\int x^2 - 3x + 4x - 12 \, dx$$

$$\int x^2 + x - 12 \, dx$$

$$\frac{x^3}{3} + \frac{x^2}{2} - 12x + c$$

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + c$$

$$4) \quad f'(x) = 6x^2 - 3x + 8$$

$$f(x) = \frac{6x^3}{3} - \frac{3x^2}{2} + 8x + c$$

$$f(x) = 2x^3 - \frac{3}{2}x^2 + 8x + c$$

$\begin{matrix} (1, 8) \\ x \quad y \end{matrix}$ lies on $f(x)$

$$8 = 2(1)^3 - \frac{3}{2}(1)^2 + 8(1) + c$$

$$c = -\frac{1}{2}$$

$$f(x) = 2x^3 - \frac{3}{2}x^2 + 8x - \frac{1}{2}$$

$$5) \quad y = 4\sqrt{x} + \frac{1}{x^2} + 10$$

$$\int y \, dx = y = 4x^{\frac{1}{2}} + x^{-2} + 10$$

$$\int y \, dx = \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + 10x + c$$

$$= \frac{8}{3}x^{\frac{3}{2}} - x^{-1} + 10x + c$$

6/

$$\int_1^3 (x+4)(x-3) dx$$

$$\int_1^3 x^2 - 3x + 4x - 12 dx$$

$$\int_1^3 x^2 + x - 12 dx$$

$$\left[\frac{x^3}{3} + \frac{x^2}{2} - 12x + c \right]_1^3$$

$$\left[\frac{(3)^3}{3} + \frac{(3)^2}{2} - 12(3) \right] - \left[\frac{(1)^3}{3} + \frac{(1)^2}{2} - 12(1) \right]$$

$$(-22.5) - \left(-\frac{67}{6}\right)$$

$$= -\frac{34}{3}$$

$$\underline{\underline{\frac{34}{3} \text{ units}^2}}$$

7

$$\frac{dy}{dx} = 10x^4 - 5$$

$$y = \frac{10x^5}{5} - 5x + c$$

$$y = 2x^5 - 5x + c \quad (2, 20)$$

$$20 = 2(2)^5 - 5(2) + c$$

$$20 = 54 + c$$

$$c = -34$$

$$\underline{y = 2x^5 - 5x - 34}$$

8)

$$\int_1^4 5 + \frac{1}{\sqrt{x}} dx$$

$$\int_1^4 5 + x^{-\frac{1}{2}} dx$$

$$\left[5x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \right]_1^4$$

$$\left[5x + 2x^{\frac{1}{2}} + C \right]_1^4$$

$$\left[5(4) + 2(4)^{\frac{1}{2}} \right] - \left[5(1) + 2(1)^{\frac{1}{2}} \right]$$

$$24 - 7$$

$$\underline{\underline{17 \text{ units}^2}}$$

9

$$f'(x) = 5 + \frac{3x^2 + 2}{x^{\frac{1}{2}}}$$

$$f'(x) = 5 + 3x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

$$f(x) = 5x + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$f(x) = 5x + \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C \quad \begin{matrix} (1, 2) \\ x \quad y \end{matrix}$$

$$2 = 5(1) + \frac{6}{5}(1)^{\frac{5}{2}} + 4(1)^{\frac{1}{2}} + C$$

$$2 = \frac{51}{5} + C$$

$$C = -\frac{41}{5}$$

$$f(x) = 5x + \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - \frac{41}{5}$$

10

$$y = x(5 - x) \quad y = 4$$

a/ intersect where:

$$4 = x(5 - x)$$

$$4 = 5x - x^2$$

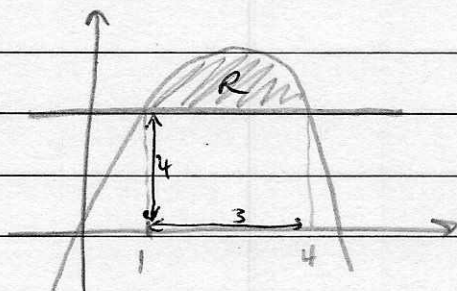
$$x^2 - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0$$

$$\underline{x = 4} \quad \underline{x = 1}$$

$$\underline{(4, 4)} \quad \text{and} \quad \underline{(1, 4)}$$

b/



Area of R = area under curve
- area of rectangle

$$\text{Area under curve} = \int_1^4 x(5 - x) dx$$

$$= \int_1^4 5x - x^2 dx$$

$$\left[\frac{5x^2}{2} - \frac{x^3}{3} + c \right]_1^4$$

$$\left[\frac{5(4)^2}{2} - \frac{(4)^3}{3} \right] - \left[\frac{5(1)^2}{2} - \frac{(1)^3}{3} \right]$$

$$\cancel{\frac{80}{2}} - \frac{13}{3} \quad \frac{56}{3} - \frac{13}{6}$$

$$\cancel{\frac{35}{6}} \quad 16.5 \text{ units}^2$$

$$\text{Area of rectangle} = 4 \times 3 = 12 \text{ units}^2$$

$$16.5 - 12 = \underline{\underline{4.5 \text{ units}^2}}$$

$$11) \quad y = x(x-2)(x-5)$$

Crosses x when $y=0$

$$0 = x(x-2)(x-5)$$

$$\underline{\underline{x=0}} \quad \underline{\underline{x=2}} \quad \underline{\underline{x=5}}$$

b/ Area of shaded region = Area between 0 and 2 + Area between 2 and 5

$$y = x(x^2 - 5x - 2x + 10)$$

$$y = x(x^2 - 7x + 10)$$

$$y = x^3 - 7x^2 + 10x$$

$$\int_0^2 x^3 - 7x^2 + 10x \, dx$$

$$\left[\frac{x^4}{4} - \frac{7x^3}{3} + \frac{10x^2}{2} + c \right]_0^2$$

$$\left[\frac{(2)^4}{4} - \frac{7(2)^3}{3} + 5(2)^2 \right] - \left[\frac{(0)^4}{4} - \frac{7(0)^3}{3} + 5(0)^2 \right]$$

$$= \underline{\underline{\frac{16}{3} \text{ units}^2}}$$

$$\left[\frac{x^4}{4} - \frac{7x^3}{3} + 5x^2 + c \right]_2^5$$

$$\left[\frac{(5)^4}{4} - \frac{7(5)^3}{3} + 5(5)^2 \right] - \left[\frac{(2)^4}{4} - \frac{7(2)^3}{3} + 5(2)^2 \right]$$

$$= \frac{-125}{12} - \frac{16}{3}$$

$$= -\frac{63}{4}$$

$$= 15.75 \text{ units}^2$$

$$15.75 + \frac{16}{3} = \frac{253}{12} \text{ units}^2$$

$$= \underline{\underline{21.1 \text{ units}^2}} \quad (3sf)$$

12a)

$$y = 10x - x^2 \quad y = 2x$$

Intersect where: $2x = 10x - x^2$

$$x^2 - 8x = 0$$

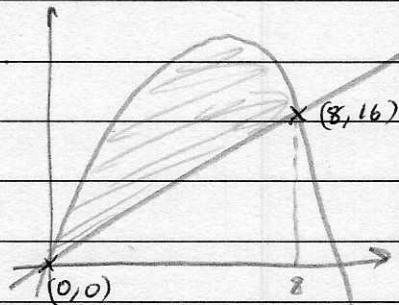
$$x(x - 8) = 0$$

$$x = 0 \quad x = 8$$

$$\text{when } x = 8 \quad y = 2(8) = 16$$

$$(\underline{0}, \underline{0}) \text{ and } (\underline{8}, \underline{16})$$

b)



Shaded Area =

Area under Curve - Area of Triangle

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}(8)(16) \\ &= \underline{64 \text{ units}^2} \end{aligned}$$

$$\int_0^8 (10x - x^2) dx$$

$$\left[5x^2 - \frac{1}{3}x^3 \right]_0^8$$

$$\left[5(8)^2 - \frac{1}{3}(8)^3 \right] - [0]$$

$$= \frac{448}{3} \text{ units}^2$$

$$\text{Shaded Area} = \frac{448}{3} - 64$$

$$= \underline{\underline{\frac{256}{3} \text{ units}^2}}$$