IYGB – FS1 PAPER N – QUESTION 1

a) REWRITE THE TABLE IN MORE USER FRIENDLY FORM

GYMNAST	A	B	C	D	E	F	G	H	I
JUDGE 1 RANK	6	4	2	1	3	5	9	8	7
JUDGE 2 RANK	7	6	4	1	2	3	8	9	5
d^2	1	4	4	0	1	4	1	1	4

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 20}{9 \times 80} = \frac{5}{6} = 0.8333$$

b)

$H_0 : \rho_s = 0$ (JUDGES ARE NOT IN GENERAL AGREEMENT)

$H_1 : \rho_s > 0$ (JUDGES ARE IN GENERAL AGREEMENT)

THE CRITICAL VALUE FOR $n=9$, AT 1% SIGNIFICANCE IS 0.7833

AS $0.8333 > 0.7833$, THERE IS EVIDENCE THAT THE JUDGES ARE IN GENERAL AGREEMENT — REJECT H_0

IYGB - FS1 PAPER M - QUESTION 2

H_0: THERE IS NO ASSOCIATION BETWEEN GENDER AND THE CLASS OF THE DEGREE ACHIEVED (INDEPENDENT EVENTS)

H_1: THERE IS ASSOCIATION BETWEEN GENDER AND THE CLASS OF THE DEGREE ACHIEVED (DEPENDENT EVENTS)

- WRITE THE TABLE WITH ACTUAL DATA

	1ST CLASS	2ND UPPER	2ND LOWER OR LESS	TOTAL
MALE	54 52.5 0.043	84 88.5 0.229	102 99 0.091	240
FEMALE	16 17.5 0.129	34 29.5 0.686	30 33 0.273	80
TOTAL	70	118	132	320

= ACTUAL DATA / OBSERVED FREQUENCY (O_i)

= EXPECTED FREQUENCY IF INDEPENDENT (E_i)

= CONTRIBUTION $\dfrac{(O_i-E_i)^2}{E_i}$

- COLLECTING THE REST OF THE INFORMATION

$$\nu = (c-1)(r-1) = (3-1)(2-1) = 2\times1 = 2$$

$$\sum_{i=1}^{6}\frac{(O_i-E_i)^2}{E_i} = 1.451$$

$$\chi^2_2(10\%) = 4.605$$

- AS $1.451 < 4.605$ THERE IS SIGNIFICANT EVIDENCE THAT THERE IS NO ASSOCIATION, SO DEAN IS JUSTIFIED — REJECT H_1

IYGB - FS1 PAPER M - QUESTION 3

- DETERMINE THE PROBABILITY SPACE DIAGRAM

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- THE PROBABILITY DISTRIBUTION OF X IS GIVEN BY

x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

- BY SYMMETRY $E(X) = 7$

- DETERMINE THE $E(X^2)$

$$E(X^2) = \left(2^2 \times \tfrac{1}{36}\right) + \left(3^2 \times \tfrac{2}{36}\right) + \left(4^2 \times \tfrac{3}{36}\right) + \cdots + \left(12^2 \times \tfrac{1}{36}\right)$$

$$= \frac{(2^2 \times 1) + (3^2 \times 2) + (4^2 \times 3) + \cdots + (12^2 \times 1)}{36}$$

$$= \frac{1974}{36} = \frac{329}{6}$$

- FINALLY THE VARIANCE

$$\text{Var}(X) = E(X^2) - \left(E(X)\right)^2$$

$$= \frac{329}{6} - 7^2 = \frac{35}{6}$$

IYGB - FS1 PAPER M - QUESTION 4

RATE OF 1.2 VISITORS PER MINUTE

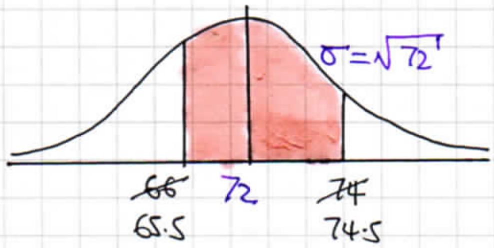
ADJUST TO ONE HOUR, $1.2 \times 60 = 72$ VISITORS PER HOUR

X = NUMBER OF VISITORS PER HOUR

$X \sim Po(72)$

APPROXIMATE BY

$Y \sim N(72, 72)$



$P(65 < X < 75)$

$= P(66 \leqslant X \leqslant 74)$

SWITCH TO A NORMAL

$= P(65.5 < Y < 74.5)$

$= P(Y < 74.5) - P(Y < 65.5)$

$= P(Y < 74.5) - [1 - P(Y > 65.5)]$

$= P(Y < 74.5) + P(Y > 65.5) - 1$

$= P\left(Z < \frac{74.5 - 72}{\sqrt{72}}\right) + P\left(Z > \frac{65.5 - 72}{\sqrt{72}}\right) - 1$

$= \Phi(0.294628\ldots) + \Phi(-0.766032\ldots) - 1$

$= 0.61586\ldots + 0.77817\ldots - 1$

$= 0.3940$

IYGB - FS1 PAPER M - QUESTION 5

a) START BY DEFINING VARIABLES AND DISTRIBUTIONS

"FISH CATCHING" RATE $\Rightarrow$ 2.5 fish per hour

ADJUSTING THE RATE TO 3 HOURS

X = NO OF FISH CAUGHT PER 3 HOURS

$X \sim Po(7.5)$

- $P(X>9) = P(X \geq 10) = 1 - P(X \leq 9) = \ldots$ tables $\ldots$

$= 1 - 0.7764 = 0.2236$

Y = NO OF DAYS (OUT OF 5), WHERE MORE THAN 9 FISH IS CAUGHT

$Y \sim B(5, 0.2236)$

- $P(Y=2) = \binom{5}{2}(0.2236)^2(0.7764)^3 = 0.234$ //

b) ADJUSTING THE RATE TO 4 HOURS — $4 \times 2.5 = 12.5$

W = NO OF FISH CAUGHT PER 4 HOURS

$W \sim Po(10)$

$H_0 : \lambda = 2.5 \ (\mu = 10)$

$H_1 : \lambda \neq 2.5 \ (\mu = 10)$

- TESTING AT 5% SIGNIFICANCE ON THE BASIS THAT $W = 16$

- $P(W \geq 16) = 1 - P(W \leq 15)$

$= 1 - 0.9513$

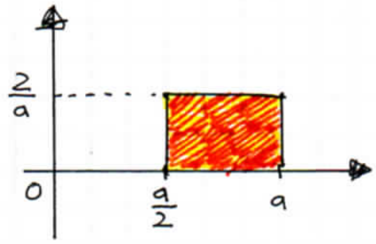
$= 0.0487 > 2.5\%$

THERE IS NO SIGNIFICANT EVIDENCE TO SUGGEST THAT THE "FISH CATCHING" RATE IS DIFFERENT — NOT SUFFICIENT EVIDENCE TO REJECT H_0 //

IYGB - FS1 PAPER M - QUESTION 6

a) ● IF X REPRESENTS THE LENGTH OF THE LONGER PIECE THEN X CAN TAKE VALUES BETWEEN $\frac{a}{2}$ & a

$$f(x) = \begin{cases} \frac{2}{a} & \frac{1}{2}a \leqslant x \leqslant a \\ 0 & \text{OTHERWISE} \end{cases}$$



● $E(X) = \int_{\frac{a}{2}}^{a} x f(x)\, dx = \int_{\frac{a}{2}}^{a} \frac{2}{a}x\, dx = \left[\frac{1}{a}x^2\right]_{a/2}^{a} = \frac{1}{a}\left[a^2 - \frac{1}{4}a^2\right]$

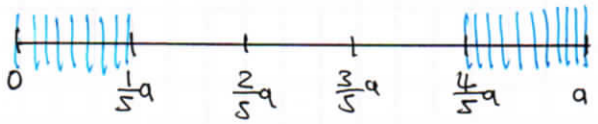
$= \frac{1}{a} \times \frac{3}{4}a^2 = \frac{3}{4}a$

● $E(X^2) = \int_{a/2}^{a} x^2 f(x)\, dx = \int_{a/2}^{a} \frac{2}{a}x^2\, dx = \left[\frac{2}{3a}x^3\right]_{a/2}^{a} = \frac{2}{3a}\left[a^3 - \frac{1}{8}a^3\right]$

$= \frac{2}{3a} \times \frac{7}{8}a^3 = \frac{7}{12}a^2$

● $\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{7}{12}a^2 - \left(\frac{3}{4}a\right)^2 = \frac{1}{48}a^2$ // AS REQUIRED

b) BY INSPECTION - DRAWING THE "ROPE" - THE CONDITION IS SATISFIED IF THE CUT IS MADE IN THE "BLUE" SECTION BELOW



∴ REQUIRED PROBABILITY IS $\frac{2}{5}$ //

ALTERNATIVE USING PART (a)

$P(X \geqslant 4(a-X)) = P(X \geqslant 4a - 4X)$

$= P(5X \geqslant 4a)$

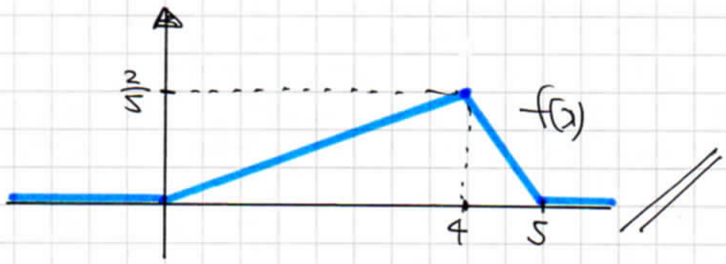
$= P(X \geqslant \frac{4}{5}a)$

$= \left(a - \frac{4}{5}a\right) \times \frac{2}{a}$

$= \frac{2}{5}$ //

IYGB - FS1 PAPER M - QUESTION 7

a) SKETCHING THE P.D.F CONSISTING OF TWO LINES



b) MODE IS 4

c) $E(X) = \int_a^b x f(x)\, dx$

$$E(X) = \int_0^4 x\left(\tfrac{1}{10}x\right) dx + \int_4^5 x\left(2 - \tfrac{2}{5}x\right) dx = \int_0^4 \tfrac{1}{10}x^2\, dx + \int_4^5 2x - \tfrac{2}{5}x^2\, dx$$

$$= \left[\tfrac{1}{30}x^3\right]_0^4 + \left[x^2 - \tfrac{2}{15}x^3\right]_4^5 = \left(\tfrac{64}{30} - 0\right) + \left(25 - \tfrac{250}{15}\right) - \left(16 - \tfrac{128}{15}\right)$$

$$= \tfrac{32}{15} + 25 - \tfrac{50}{3} - 16 + \tfrac{128}{15} = 3$$

AS REQUIRED

d) FIRST COMPUTE $E(X^2) = \int_a^b x^2 f(x)\, dx$

$$E(X^2) = \int_0^4 x^2\left(\tfrac{1}{10}x\right) dx + \int_4^5 x^2\left(2 - \tfrac{2}{5}x\right) dx = \int_0^4 \tfrac{1}{10}x^3\, dx + \int_4^5 2x^2 - \tfrac{2}{5}x^3\, dx$$

$$= \left[\tfrac{1}{40}x^4\right]_0^4 + \left[\tfrac{2}{3}x^3 - \tfrac{1}{10}x^4\right]_4^5$$

$$= \left(\tfrac{32}{5} - 0\right) + \left(\tfrac{250}{3} - \tfrac{125}{2}\right) - \left(\tfrac{128}{3} - \tfrac{128}{5}\right) = \tfrac{61}{6}$$

USING $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$

$$\mathrm{Var}(X) = \tfrac{61}{6} - 3^2$$

$$\mathrm{Var}(X) = \tfrac{7}{6}$$

IYGB

$$F(x) = \int_a^x f(x)\,dx$$

$$F_1(x) = \int_0^x \frac{1}{10}x\,dx = \left[\frac{1}{20}x^2\right]_0^x = \frac{1}{20}x^2 - 0 = \frac{1}{20}x^2$$

$$F_1(4) = \frac{4}{5}$$

$$F_2(x) = \frac{4}{5} + \int_4^x 2 - \frac{2}{5}x\,dx = \frac{4}{5} + \left[2x - \frac{1}{5}x^2\right]_4^x$$

$$= \frac{4}{5} + \left[2x - \frac{1}{5}x^2\right] - \left[8 - \frac{16}{5}\right]$$

$$= \frac{4}{5} + 2x - \frac{1}{5}x^2 - 8 + \frac{16}{5}$$

$$= -\frac{1}{5}x^2 + 2x - 4$$

SPECIFYING

$$\begin{cases} 0 & x<0 \\ \frac{1}{20}x^2 & \leqslant x < 4 \\ -\frac{1}{5}(x^2-10x+20) & 4 \leq x < 5 \\ 1 & x > 5 \end{cases}$$