# Exponentials and Logarithms

Exponentials and logs might seem a bit tricky at first, but once you get used to them and they get used to you, you'll wonder what you ever worried about. Plus, they're really handy for modelling real-life situations.

**1** Given that $\log_a x = \log_a 4 + 3\log_a 2$, show that $x = 32$.

**(2 marks)**

**2** Given that $p > 0$, what is the value of $\log_p(p^4) + \log_p(\sqrt{p}) - \log_p\left(\frac{1}{\sqrt{p}}\right)$?

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**(3 marks)**

**3** It is given that $2^x = 9$.

**a)** Find the value of $x$, giving your answer to 2 decimal places.

$x =$ .....

**(2 marks)**

**b)** Hence, or otherwise, solve the equation $2^{2x} - 13(2^x) + 36 = 0$, giving each solution to an appropriate degree of accuracy.

This should remind you of a quadratic equation...

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**(5 marks)**

**4** Solve the equation $3^{(y^2-4)} = 7^{(y+2)}$, giving your answers to 3 significant figures where appropriate.

Start by taking logs of both sides.

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**(5 marks)**

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**5** For the positive integers $p$ and $q$, $\log_4 p - \log_4 q = \frac{1}{2}$.

**a)** Show that $p = 2q$.

**(3 marks)**

**b)** The values of $p$ and $q$ are such that $\log_2 p + \log_2 q = 7$.
Use this information to find the values of $p$ and $q$.

$p =$ ..... $q =$ .....

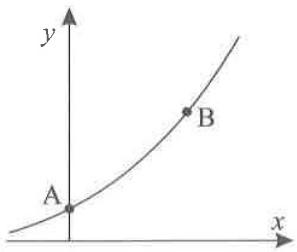
**(5 marks)**

**6** For the function $f(x) = 3 \ln x - \ln 3x$, $x > 0$, find the exact value of $x$ when $f(x) = 0$.

$x =$ .....

**(2 marks)**

**7** The curve below has equation $y = e^{kx}$.




**a)** Write down the coordinates of A.

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**(1 mark)**

**b)** The gradient of the curve at A is 4.
Write down the value of $k$.

$k =$ .....

**(1 mark)**

**c)** Find the exact gradient of the curve at the point where $x = -1$.

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**(1 mark)**

**d)** The gradient of the curve at B is $4e^8$. Find the coordinates of B.

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**(2 marks)**

# Exponentials and Logarithms

**8** The UK population, $P$, of an endangered species of bird is modelled over time, $t$ years ($t \geq 0$), by the function: $P = 5700e^{-0.15t}$. The UK population, $Q$, of a bird of prey that hunts the endangered species, as well as other animals, is modelled by the function: $Q = 2100 - 1500e^{-0.15t}$. The time $t = 0$ represents the beginning of the year 2010, when the bird of prey was first introduced into the country.

**a)** Find the year in which the population of the bird of prey is first predicted to exceed the population of the endangered species according to these models.

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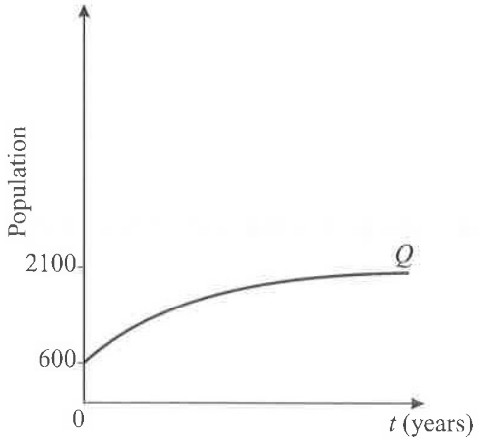
**(4 marks)**

**b)** The graph showing the predicted population of the bird of prey is shown on the right. Add a curve to the graph to show the predicted UK population of the endangered species of bird over the same time period.




**(2 marks)**

**c)** Comment on the validity of each population model.

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**(2 marks)**

**d)** Predict the year that the population of the endangered species will drop to below 1000.

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**(2 marks)**

**e)** When this population drops below 1000, conservationists start enacting a plan to save the species. Suggest one refinement that could be made to the model to take this into account.

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**(1 mark)**

# Exponentials and Logarithms

**9** The number of supporters of a local football team has tended to increase in recent years. The following table shows the average home game attendance for the club in recent seasons.

| Season | 11/12 | 12/13 | 13/14 | 14/15 | 15/16 |
|---|---|---|---|---|---|
| Attendance (in hundreds, to the nearest hundred) | 1 | 2 | 4 | 8 | 14 |

The attendance can be modelled by an equation of the form $y = ab^t$, where $y$ is the average home game attendance in hundreds, $t$ is the number of years after the 2010/11 season, and $a$ and $b$ are constants to be found.

**a)** Show that $y = ab^t$ can be written in the form $\log_{10} y = t\log_{10} b + \log_{10} a$.

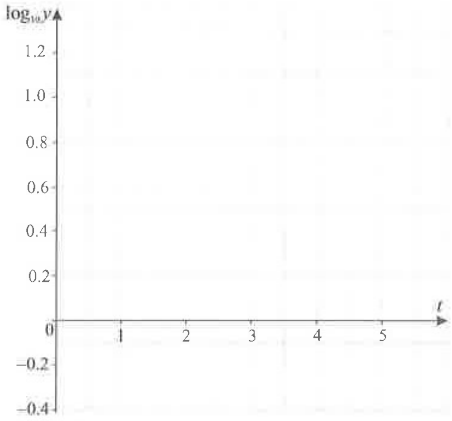
**(2 marks)**

**b)** Complete the following table, giving values correct to 3 d.p.:

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\log_{10} y$ | 0 | 0.301 | | | |

**(1 mark)**

**c)** Plot the graph of $\log_{10} y$ against $t$. Draw by eye a line of best fit for your data and calculate the values of $a$ and $b$.




$a$ = ..... $b$ = .....

**(4 marks)**

**d)** The chairman has promised a new stadium for the club when the average home game attendance exceeds 5000. According to this model, in which season will the attendance exceed this value?

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**(3 marks)**

EXAM TIP

If you're asked to comment on the validity of models, or asked for limitations or refinements, think about anything that stops the model being realistic. For example, it might predict that a quantity will continue to rise until it becomes infinitely big, which might not be likely in a real-life situation — so you could refine the model by introducing an upper limit.

Score

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