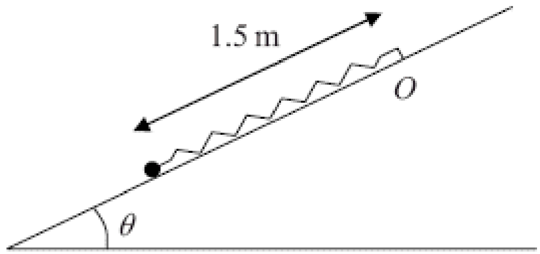
1.



A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in the diagram above. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(Total 9 marks)

2. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(a) Show that $v^2 = 2g(a+x) - \dfrac{3gx^2}{2a}$.

(4)

(b) Find the greatest speed attained by P as it falls.

(4)

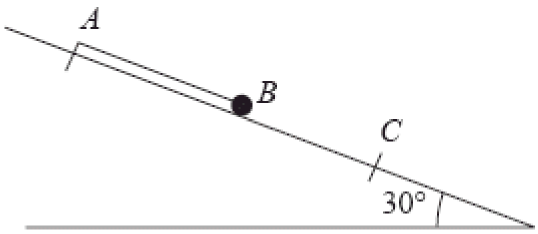
After release, P next comes to instantaneous rest at a point D.

(c) Find the magnitude of the acceleration of P at D.

(6)

(Total 14 marks)

3.



One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in the diagram above.
Find

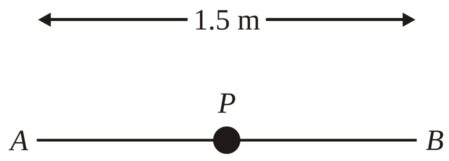
(a) the length AC,

(5)

(b) the greatest speed attained by B as it moves from its initial position to C.

(7)
(Total 12 marks)

4. Two light elastic strings each have natural length 0.75 m and modulus of elasticity 49 N. A particle P of mass 2 kg is attached to one end of each string. The other ends of the strings are attached to fixed points A and B, where AB is horizontal and AB = 1.5 m.



The particle is held at the mid-point of AB. The particle is released from rest, as shown in the figure above.

(a) Find the speed of P when it has fallen a distance of 1 m.

(6)

Given instead that P hangs in equilibrium vertically below the mid-point of AB, with $\angle APB = 2\alpha$,

(b) show that $\tan\alpha + 5\sin\alpha = 5$.

(6)
(Total 12 marks)

5. In a "test your strength" game at an amusement park, competitors hit one end of a small lever with a hammer, causing the other end of the lever to strike a ball which then moves in a vertical tube whose total height is adjustable. The ball is attached to one end of an elastic spring of natural length 3 m and modulus of elasticity 120 N. The mass of the ball is 2 kg. The other end of the spring is attached to the top of the tube. The ball is modelled as a particle, the spring as light and the tube is assumed to be smooth.

The height of the tube is first set at 3 m. A competitor gives the ball an initial speed of 10 m s^{-1}.

(a) Find the height to which the ball rises before coming to rest.

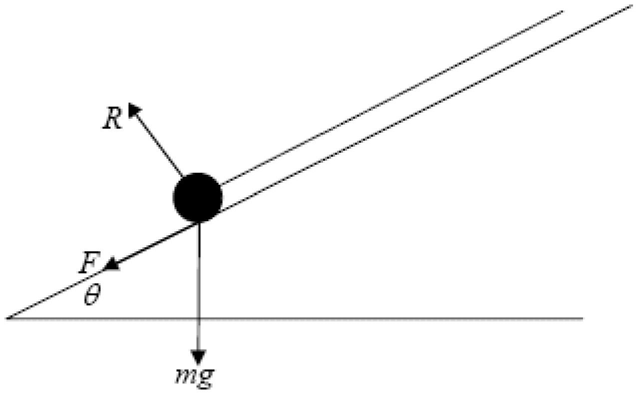
(6)

The tube is now adjusted by reducing its height to 2.5 m. The spring and the ball remain unchanged.

(b) Find the initial speed which the ball must now have if it is to rise by the same distance as in part (a).

(5)
(Total 11 marks)

1.



EPE lost $= \dfrac{\lambda \times 0.6^2}{2 \times 0.9} - \dfrac{\lambda \times 0.1^2}{2 \times 0.9} \left(= \dfrac{7}{36}\lambda \right)$ M1 A1

R$(\uparrow)$ $R = mg\cos\theta$ M1

$= 0.5g \times \dfrac{4}{5} = 0.4g$

$F = \mu R = 0.15 \times 0.4g$ M1 A1

P.E. gained = E.P.E. lost − work done against friction

$0.5\text{g} \times 0.7\sin\theta = \dfrac{\lambda \times 0.6^2}{2 \times 0.9} - \dfrac{\lambda \times 0.1^2}{2 \times 0.9} - 0.15 \times 0.4g \times 0.7$ M1 A1 A1

$0.1944\lambda = 0.5 \times 9.8 \times 0.7 \times \dfrac{3}{5} + 0.15 \times 0.4 \times 9.8 \times 0.7$

$\lambda = 12.70......$

$\lambda = 13$ N or 12.7 A1

[9]

2. (a) $\dfrac{1}{2}mv^2 + \dfrac{3mgx^2}{4a} = mg(a + x)$ M1 A2 (1, 0)

leading to $v^2 = 2g(a + x) - \dfrac{3gx^2}{2a}$ * cso A1 4

(b) Greatest speed is when the acceleration is zero

$$T = \frac{\lambda x}{a} = \frac{3mgx}{2a} = mg \Rightarrow x = \frac{2a}{3}$$ M1 A1

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$ M1

$$v = \frac{2}{3}\sqrt{(6ag)}$$ accept exact equivalents A1 4

<u>Alternative</u>

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}$$

Differentiating with respect to *x*

$$2v\frac{\mathrm{d}v}{\mathrm{d}x} = 2g - \frac{3gx}{a}$$

$$\frac{\mathrm{d}v}{\mathrm{d}x} = 0 \Rightarrow x = \frac{2a}{3}$$ M1 A1

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$ M1

$$v = \frac{2}{3}\sqrt{(6ag)}$$ accept exact equivalents A1

(c) $v = 0 \Rightarrow 2g(a + x) - \dfrac{3gx^2}{2a} = 0$ M1

$3x^2 - 4ax - 4a^2 = (x - 2a)(3x + 2a) = 0$

$x = 2a$ M1 A1

At D, $m\ddot{x} = mg - \dfrac{\lambda \times 2a}{a}$ ft their $2a$ M1 A1ft

$|\ddot{x}| = 2g$ A1 6

Alternative approach using SHM for (b) and (c)

If SHM is used mark (b) and (c) together placing the marks in the gird as shown.

Establishment of equilibrium position

$$T = \frac{\lambda x}{a} = \frac{3mge}{2a} = mg \Rightarrow e = \frac{2a}{3}$$ bM1 bA1

N2L, using y for displacement from equilibrium position

$$m\ddot{y} = mg - \frac{\frac{3}{2}mg(y+e)}{a} = -\frac{3g}{2a}y$$ bM1 bA1

$$\omega^2 = \frac{3g}{2a}$$

Speed at end of free fall $u^2 = 2ga$ cM1

Using A for amplitude and $v^2 = \omega^2 (a^2 - x^2)$

$$u^2 = 2ga \text{ when } y = -\frac{2}{3}a \Rightarrow 2ga = \frac{3g}{2a}\left(A^2 - \frac{4a^2}{9}\right)$$ cM1

$$A = \frac{4a}{3}$$ cA1

Maximum speed $A\omega = \frac{4a}{3} \times \sqrt{\left(\frac{3g}{2a}\right)} = \frac{2}{3}\sqrt{(6ag)}$ cM1 cA1

Maximum acceleration $A\omega^2 = \frac{4a}{3} \times \frac{3g}{2a} = 2g$ cA1

[14]

3. (a) Let x be the distance from the initial position of B to C

GPE lost = EPE gained

$$mgx \sin 30° = \frac{6mgx^2}{2a}$$ M1 A1 = A1

Leading to $x = \frac{a}{6}$ M1

$$AC = \frac{7a}{6}$$ A1 5

(b) The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal.

(↖) $T = mg\sin 30° = \dfrac{6\text{mge}}{\text{a}}$ M1

$e = \dfrac{a}{12}$ A1

CE $\dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}\left(\dfrac{a}{12}\right)^2 = mg\dfrac{a}{12}\sin 30°$ M1 A1 = A1

Leading to $v = \sqrt{\left(\dfrac{ga}{24}\right)} = \dfrac{\sqrt{6ga}}{12}$ M1 A1 7

Alternative approach to (b) *using calculus with energy.*

Let distance moved by *B* be *x*

CE $\dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}x^2 = mgx\sin 30°$ M1 A1 = A1

$$v^2 = gx - \frac{6g}{a}x^2$$

For maximum *v* $\dfrac{\mathrm{d}}{\mathrm{d}x}(v^2) = 2v\dfrac{\mathrm{d}v}{\mathrm{d}x} = g - \dfrac{12g}{a}x = 0$ M1 A1

$$x = \frac{a}{12}$$

$v^2 = g\left(\dfrac{a}{12}\right) - \dfrac{6g}{a}\left(\dfrac{a}{12}\right)^2 = \dfrac{ga}{24}$ M1

$v = \sqrt{\left(\dfrac{ga}{24}\right)}$ A1 7

Alternative approach to (b) *using calculus with Newton's second law.*

As before, the centre of the oscillation is when extension is $\frac{a}{12}$ M1 A1

N2L $mg\sin 30° - T = m\ddot{x}$

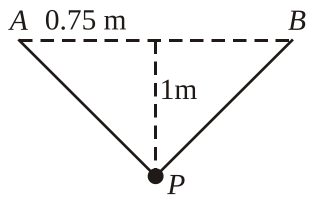
$$\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12}+x\right)}{a} = m\ddot{x}$$ M1 A1

$$\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$$ A1

$$v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$$ M1 A1 7

[12]

4. (a)



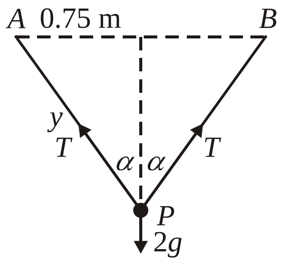
$AP = \sqrt{(0.75^2 + 1^2)} = 1.25$ M1 A1

Conservation of energy

$$\frac{1}{2}\times 2\times v^2 + 2\times\frac{49\times 0.5^2}{2\times 0.75} = 2g\times 1$$ −1 for each incorrect term M1 A2 (1, 0)

Leading to $v \approx 1.8$ (ms^{-1}) accept 1.81 A1 6

(b)



R (↑) $2T\cos\alpha = 2g$ M1 A1

$$y = \frac{0.75}{\sin\alpha}$$

Hooke's Law $T = \frac{49}{0.75}\left(\frac{0.75}{\sin\alpha} - 0.75\right)$ M1 A1

$$= 49\left(\frac{1}{\sin\alpha} - 1\right)$$

$\frac{9.8}{\cos\alpha} = 49\left(\frac{1}{\sin\alpha} - 1\right)$ Eliminating T M1

$$\tan\alpha = 5(1 - \sin\alpha)$$

$5 = \tan\alpha + 5\sin\alpha$ cso A1 6

[12]

5. (a) Energy: $\frac{1}{2} \times 2 \times 10^2 = 2 \times 9.8 \times h + \frac{1}{2} \times \frac{120 \times h^2}{3}$ M1 A1 A1

$20h^2 + 19.6h - 100 = 0$ M1

$h = \frac{-19.6 \pm \sqrt{(19.6^2 + 4 \times 20 \times 100}}{40}$ M1

$= 1.7991\ldots \approx 1.8$ (or 1.80) m A1 6

(b) $\frac{1}{2} \times 2 \times V^2 = 2 \times 9.8 \times 1.8 + \frac{1}{2} \times \frac{120 \times 2.3^2}{3} - \frac{1}{2} \times \frac{120 \times 0.5^2}{3}$ M1 A1 A1

$V = 11.7$ (3 s.f.) or 12 (2 s.f.) m s^{-1} M1 A1 5

[11]

1. This was probably the least well done of all the questions and correct solutions were relatively rare. The most common mistake was the assumption that there was no final EPE but this was often combined with other errors to give a huge variety of different wrong answers. A surprisingly large proportion of candidates treated this as an equilibrium question, either starting with $T = \mu R + mg \sin \theta$ or slipping an EPE term in as well for good measure. Others realised that it was an energy question but forgot to include the work done against friction; these attempts either used only the frictional force in their equation or ignored it completely, offering as their solution "Initial EPE = mgh". Another common error was to include the GPE term twice, once as energy and again as part of the "Work done" expression, showing a lack of understanding of the origin of the mgh formula. Very many candidates scored only the 3 marks for finding friction, while those who thought that this was a simple conversion of EPE into GPE had no need to find the friction and so didn't even earn these. Some candidates who included all necessary terms fell at the accuracy hurdle. Inexplicably, a final extension/ compression of 0.2 was not uncommon and other errors arose from inappropriate use of the various lengths mentioned, 1.5, 0.9 and 0.7. There were also all the usual sign errors generated by mistakes in identifying gains and losses. A few candidates produced a perfect solution but lost the final mark by giving their answer as 12.7008.

2. Most candidates managed to arrive at the required result in part (a), though some unnecessarily split the motion into two parts, considering freefall initially to find the kinetic energy when the string became taut and then proceeding to consider the taut string and others would clearly have failed had not the answer been provided.

Parts (b) and (c) were often difficult to disentangle. Some candidates took an SHM approach from the start of to the Examiner". The main fault was not when to start considering SHM but not establishing a correct equation to prove that the motion was SHM; no credit is given for making assumptions of this nature. A fully correct solution using SHM was rare, the equations frequently being unsatisfactory due to using x for the distance from the equilibrium point and confusing it with x as defined in the question to be the extension of the string.

For the non-SHM solutions, in part (b) many candidates assumed that the maximum speed occurred when $x = 0$ rather than when $a = 0$. In part (c) most substituted $v = 0$ in the result from part (a). Some did not expect to obtain a quadratic and so stopped working (or ran out of time?). Of those who obtained a solution for their quadratic equation, some would then try incorrectly to use their value for x as the amplitude in SHM instead of using an equation of motion and Hooke's law. Many equations of motion omitted the weight of the particle.

3. Many candidates used unduly complicated methods for both parts of this question. Some tried to use S.H.M. but very few attempted to establish the motion as being S.H.M. before quoting and using the standard formulae. In part (a) there was confusion between equilibrium and rest positions. Some candidates used Hooke's law and resolved parallel to the plane, finding the equilibrium extension and claimed that the particle was at rest at this point. They then proceeded to use the same extension (now correctly) in (b) to obtain the greatest speed. Energy methods often had a missing term or a gravitational potential energy term which was inconsistent with the positions involved. Alternative methods using Newton's second law and calculus were reasonably popular but many were incomplete.

4. Part (a) was well done. The only common error was considering the elastic potential energy in only one part of the string instead of in both parts. Most candidates realised that energy was involved and the few who attempted using Newton's Second Law almost all failed to consider a general point of the motion and so gained no credit. Nearly all candidates could start part (b) by resolving vertically and writing down some form of Hooke's Law. The manipulations required to obtain the required trigonometric relation, however, were demanding and even strong candidates often needed two or three attempts to complete this and the time spent on this was sometimes reflected in an inability to complete the paper. This was particular the case if candidates attempted to use or gain information by writing down an equation of energy. This leads to very complicated algebra and is not a practical method of solving questions of this type at this level. (Correctly applied it leads to a quartic not solvable by elementary methods.) For those who were successful in part (a), writing T in terms of , say, the angle made by each part of the string with the vertical proved the critical step. If they obtained $T = \frac{49}{0.75}\left(\frac{0.75}{\sin a} - 0.75\right)$, or its equivalent, the majority of candidates had the necessary trigonometric skills to complete the question.

5. No Report available for this question.