Worksheet 9 Solutions

Question 1 Solution.

$$\begin{aligned}\int (2p-4)^2 dp &= \int (4p^2 - 16p + 16) dp \\ &= \frac{4}{3}p^3 - \frac{16}{2}p^2 + 16p + c \\ &= \frac{4}{3}p^3 - 8p^2 + 16p + c\end{aligned}$$

Question 2 Solution.

(a) *Counterexample.* For example, let $p = 7$, then p is an odd prime, but $p + 2 = 9$ which is not a prime (since it is divisible by 3).

(b) *Counterexample.* For example, let $x = 120$, then $\sin(120 + 60) = \sin(180) = 0$, which is not positive. [NB: be careful of the interval given: you cannot let $x = 180$ as your counter-example.]

(c) *Counterexample.* For example, let $x = 0$ and $x = 90$, then

$$\sin(x + y) = \sin(90) = 1$$

but

$$\sin(x)\sin(y) = \sin(0)\sin(90) = 0(1) = 0 \neq 1$$

therefore $\sin(x + y) \neq \sin(x)\sin(y)$ for all x and y.

(d) *Counterexample.* For example, let $f(x) = x$ and $g(x) = x^2$. Then the derivative of their product is

$$\frac{d}{dx}(x \times x^2) = \frac{d}{dx}(x^3) = 3x^2$$

while the product of their derivatives is

$$\frac{d}{dx}(x) \times \frac{d}{dx}(x^2) = 1 \times 2x = 2x$$

and $2x$ and $3x^2$ are (generally) not equal.

Question 3 Solution.

(a) To find the equation of the line l_2, we need its gradient and a point it goes through.

Let's work on a point it passes through first: it is a normal to the curve $y = x^{-1}+2x^2+x(1-x)$ at $x = 1$. The y-coordinate on the curve when $x = 1$ is

$$y = 1^{-1} + 2(1)^2 + 1(1-1) = 1 + 2 = 3$$

so the line l_2 passes through the point $(1, 3)$.

Now let's work on finding its gradient. First let's find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -x^{-2} + 4x + 1 - 2x$$

so the gradient of the curve at $x = 1$ is

$$\begin{aligned} \left.\frac{dy}{dx}\right|_{x=1} &= -(1)^{-2} + 4(1) + 1 - 2(1) \\ &= -1 + 4 + 1 - 2 \\ &= 2 \end{aligned}$$

so the gradient of the normal to the curve at $x = 1$ is $-\frac{1}{2}$.

Putting this all together, we find that the equation of l_2 is:

$$\begin{aligned} &y - 3 = -\frac{1}{2}(x-1) \\ &\Rightarrow 2y - 6 = -x + 1 \\ &\Rightarrow y = -\frac{1}{2}x + \frac{7}{2} \end{aligned}$$

(b) The coordinates of P are $\boxed{\left(0, \frac{7}{2}\right)}$.

To find the coordinates of Q, we solve the equations of l_1 and l_2 simultaneously:

$$\begin{aligned} &x - 4 = -\frac{1}{2}x + \frac{7}{2} \\ &\Rightarrow 2x - 8 = -x + 7 \\ &\Rightarrow x = 5 \end{aligned}$$

and so $y = 1$. Therefore the coordinates of Q are $\boxed{(5,1)}$.

(c) The coordinates of R are $(4, 0)$.

From a diagram (which you should draw), you can find that the area of $OPQR$ can be given by

$$\text{Area of } QPQR = \left(\frac{3.5+1}{2} \times 5\right) - \left(\frac{1 \times 1}{2}\right) = \frac{43}{4}$$

Note: there are other expressions possible. Here we have dropped a perpendicular line from Q to the x-axis, found the area of the trapezium and subtracted the area of the triangle. You can also consider the trapezium formed by OPQ and the point when $x = 4$ on l_2 and add its area to the area of the remaining triangle.

Question 4 Solution.

(a) The time taken for the particle to move from A to B is given by

$$2 = 8t - \frac{1}{2}t^2 \Rightarrow t^2 - 16t + 4 = 0$$

which has two solutions $t = 0.254...$ and $t = 15.745...$. Now this is where context is important as it tells us to ignore the second value of t, since once the particle arrives at B a first time (at $t = 0.254...$), the situation changes.

So the total time for the particle to move from A to C is $0.254... + 5 = \boxed{5.25 \text{ s}}$.

(b) To find distance travelled by the particle between B and C, we need the speed of the particle at B. This is given by

$$v^2 = 8^2 + 2(-1)(2) \Rightarrow v = 7.745...$$

so the distance travelled by the particle between B and C is $(7.745...)(5) = 38.729...$.

So the total distance travelled by the particle between A and C is $2 + 38.729... = \boxed{40.73 \text{ m}}$.

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