KS5 "Full Coverage": Modelling with Exponential Equations

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Find the 'initial' value using an exponential model.

[Edexcel C3 June 2015 Q4a Edited]

Water is being heated in an electric kettle. The temperature, θ° C, of the water t seconds after the kettle is switched on, is modelled by the equation

$\theta = 120 - 100e^{-\lambda t}$, $0 \leq t \leq T$.

Find the initial temperature of the water.

$\theta =$ .... $^{\circ}$ C

Question 2

Categorisation: Substitute a value into an exponential model.

[Edexcel A2 Specimen Papers P2 Q3b Edited]

A cup of hot tea was placed on a table. At time t minutes after the cup was placed on the table, the temperature of the tea in the cup, θ

$$\theta = 25 + 50e^{-0.03t}$$

.... minutes

Question 3

Categorisation: Determine the time required to give a certain output in an exponential model.

[Edexcel AS Specimen Papers P1 Q13c]

The growth of pond weed on the surface of a pond is being investigated.
The surface area of the pond covered by the weed, A m^2, can be modelled by the equation

$$A = 0.2e^{0.3t}$$

where t is the number of days after the start of the investigation.

Given that the pond has a surface area of 100 m^2, find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed.

.... days

and .... hours

Question 4

Categorisation: Use a given input and output to determine the value of a constant in an exponential model.

[Edexcel C3 Jan 2011 Q4b Edited] Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of th9e tea is modelled by the equation

$$\theta = 20 + 70e^{-kt}$$

where k is a positive constant.
The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

Show that $k = p \ln q$ where p and q are positive constants to be found.

....

Question 5

Categorisation: Deal with the gradient function of an exponential model.

[Edexcel C3 June 2011 Q5c Edited]

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = 7.5e^{-kt}$$

where k is a positive constant.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

Given that $= \frac{1}{4}\, ln\, 3$, find the value of t when $\frac{dm}{dt} = -0.6\, ln\, 3$.

Give your answer correct to 1 decimal place.

……………………

Question 6

Categorisation: As above, but where the base of the power is not e. [Recall that $\frac{d}{dx}(a^x) = a^x\, ln\, a$]

[Edexcel C4 Jan 2011 Q2] The current, I amps, in an electric circuit at time t seconds is given by

$I = 16 - 16(0.5)^t$, $t \geq 0$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $ln\, a$, where a is a constant.

……………………

Question 7

Categorisation: Use the value after some time has elapsed to provide the initial value of a second instance of the exponential model.

[Edexcel C3 June 2007 Q8c] The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t}$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given. A second dose of 10 mg is given after 5 hours. No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg. Find the value of T .

$T =$ ....

Question 8

Categorisation: Determine the 'long term value' of an exponential model.

[Edexcel C3 June 2013(R) Q8b]

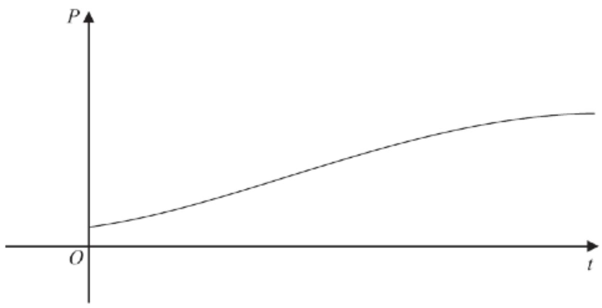


Figure 3

The population of a town is being studied. The population *P*, at time *t* years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, t \geq 0$$

where k is a positive constant. The graph of *P* against *t* is shown in Figure 3. Use the given equation to find a value for the expected upper limit of the population.

....

Question 9

Categorisation: Determine the equation of an exponential model using two points on the graph.

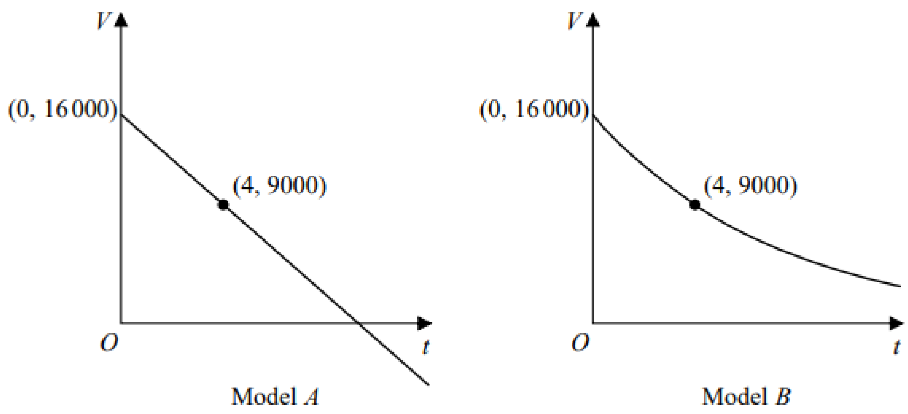
[Edexcel A2 SAM P1 Q6bi] A company plans to extract oil from an oil field.

The daily volume of oil V , measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



Using an exponential model and the information given in the question, find a possible equation for model B.

$V =$ ....

Question 10

Categorisation: Describe the limitations of linear or exponential models.

(Continued from previous question)

Write down a limitation of using model A.

....

Question 11

Categorisation: Interpret the meaning of a in $y = ab^t$

[Edexcel A2 Specimen Papers P2 Q7di Edited]

A bacterial culture has area p mm ^{2} at time t hours after the culture was placed onto a circular dish.

A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

It can be shown that the scientist's model for p leads to the equation $p = ae^{kt}$ where a and k are constants.

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of $ln\ p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

It can be shown that $a = 52$ and $k = 0.14$ to 2 significant figures.

The model for p can be rewritten as

$$p = ab^t$$

Iinterpret the value of the constant a .

....

Question 12

Categorisation: Interpret the meaning of b in $y = ab^t$

(Continued from previous question)

Iinterpret the value of the constant b.

……………………

Question 13

Categorisation: Perform exponential regression, i.e. log an exponential model to obtain a linear model, find the gradient/y-intercept, then use these to determine the original constants in the exponential model.

[Edexcel A2 Specimen Papers P2 Q7b Edited]

A bacterial culture has area p mm ^{2} at time t hours after the culture was placed onto a circular dish. A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

It can be shown that the scientist's model for p leads to the equation $p = ae^{kt}$ where a and k are constants.

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of $ln\ p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

Estimate, to 2 significant figures, the value of a and the value of k .

……………………

Question 14

Categorisation: Rewrite e^{kt} in the form b^t (i.e. with no coefficient on the exponent)

(Continued from previous question)

Hence show that the model for p can be rewritten as

$$p = ab^t$$

stating, to 3 significant figures, the value of the constant b .

....

Question 15

Categorisation: Further practice of exponential regression.

[Edexcel AS SAM P1 Q14a]

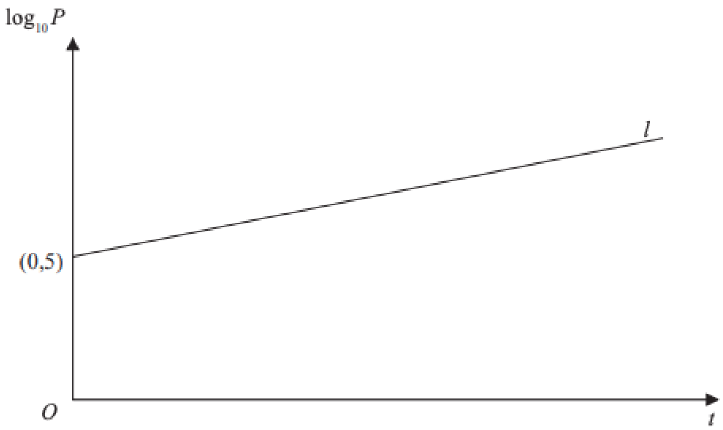


Figure 2

A town's population, P , is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $log_{10}P$ for the population over a period of 100 years.

The line l meets the vertical axis at $(0,5)$ as shown. The gradient of l is $\frac{1}{200}$. Write down an equation for l .

$log_{10}P =$ ....

Question 16

Categorisation: As above, but with polynomial models.

[Edexcel A2 SAM P1 Q12a] In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form
$N = aT^b$, where a and b are constants
Show that this relationship can be expressed in the form

$$log_{10}N = m\, log_{10}T + c$$

giving m and c in terms of the constants a and/or b.

....

Question 17

Categorisation: Determine the constants in a polynomial model.

[Edexcel A2 SAM P1 Q12b Edited] In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form
$N = aT^b$, where a and b are constants
It can be shown that: $log_{10}N = b\, log_{10}T + log_{10}a$

Figure 3 shows the line of best fit for values of $log_{10}N$ plotted against values of $log_{10}T$

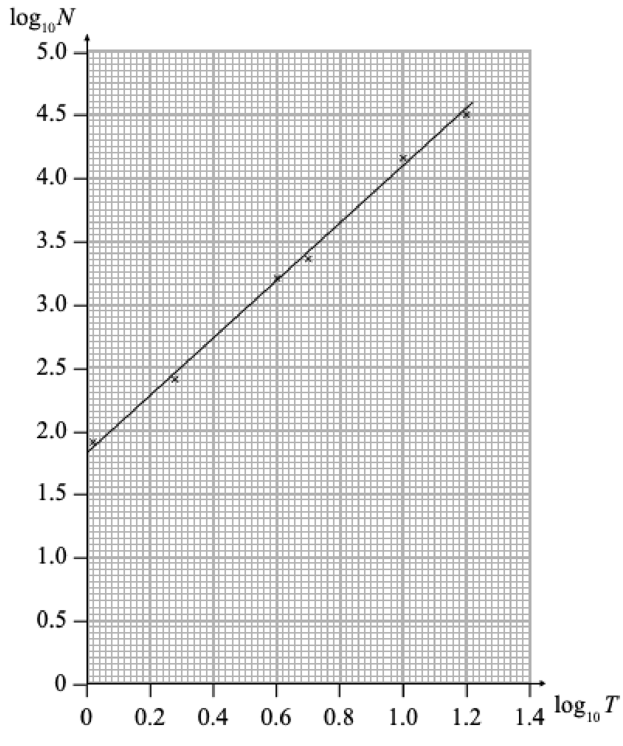


Figure 3

Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

.... microbes

Answers

Question 1

$\theta = 20\ ^{\circ}\mathrm{C}$

Question 2

11.9 minutes

Question 3

20 days and and 17 hours

Question 4

$p = \frac{1}{5}, q = 2$

Question 5

$t = 4.1$

Question 6

$ln\ 4$

Question 7

$T = 13$

Question 8

8000

Question 9

$V = 16000e^{\frac{1}{4} ln\left(\frac{9}{16}\right)t}$

Question 10

The graph shows that the volume will become negative

Question 11

The initial area of bacteria.

Question 12

The rate at which the area of bacteria is increasing per minute (e.g. 1.1 represent a 10% increase)

Question 13

$a = 52, k = 0.14$

Question 14

$b = 1.15$

Question 15

$log_{10}P = \frac{1}{200}t + 5$

Question 16

$m = b, c = log_{10}a$

Question 17

800 microbes