Q1.

Given that $\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $\mathbf{M}^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ for all $n \in \mathbb{N}$

(Total 5 marks)

Q2.

Prove by induction that

$$\frac{2\times 1}{2\times 3}+\frac{2^2\times 2}{3\times 4}+\frac{2^3\times 3}{4\times 5}+\ldots+\frac{2^n\times n}{(n+1)(n+2)}=\frac{2^{n+1}}{n+2}-1$$

for all integers $n \geq 1$.

(Total 7 marks)

Q3.

The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2, \qquad u_{n+1} = \frac{5u_n - 3}{3u_n - 1}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{3n+1}{3n-1}$$

(Total 6 marks)

Q4.

(a) Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(4)

(b) **Hence** show that

$$\sum_{r=1}^{2n} r(r-1)(r+1) = n(n+1)(2n-1)(2n+1)$$

(4)
(Total 8 marks)

Q5.

Prove that $8^n - 7n + 6$ is divisible by 7 for all integers $n \geq 0$

(Total 5 marks)