Exam Questions – Statistical Distributions

1. The random variable X has probability function

$$P(X = x) = \frac{(2x-1)}{36} \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of X.

(3)

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Find

(b) $P(2 < X \leq 5)$,

$$\frac{5}{36} + \frac{7}{36} + \frac{9}{36} = \frac{21}{36} \text{ or } \frac{7}{12} \text{ or } 0.583$$

(2)
(Total 5 marks)

2. A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.

(a) Find the probability that Linda scores 30 points in a round.

(2)

$$0.6 \times 0.6 \times 0.6 = 0.216$$

The random variable X is the number of points Linda scores in a round.

(b) Find the probability distribution of X.

(5)

x	0	10	20	30
$P(X=x)$	0.4	$0.6 \times 0.4 = 0.24$	$0.6 \times 0.6 \times 0.4 = 0.144$	$0.6 \times 0.6 \times 0.6 = 0.216$

(Total 7 marks)

3. The random variables R, is distributed as follows

$$R \sim B(15, 0.3),$$

Find $P(R = 5)$,

$$^{15}C_5 (0.3)^5 (0.7)^{10} = 0.20613$$

(2)
(Total 2 marks)

4. A bag contains a large number of counters of which 15% are coloured red. A random sample of 30 counters is selected and the number of red counters is recorded.

Find the probability of no more than 6 red counters in this sample.

$X \sim B(30, 0.15)$

$P(X \leq 6) = 0.8474$ ← table

(2)
(Total 2 marks)

5. A farmer noticed that some of the eggs laid by his hens had double yolks. He estimated the probability of this happening to be 0.05. Eggs are packed in boxes of 12.

Find the probability that in a box, the number of eggs with double yolks will be

(a) exactly one,

$X \sim B(12, 0.05)$

$P(X=1) = {}^{12}C_1 (0.05)^1 (0.95)^{11} = 0.341$

(3)

(b) more than three.

$P(X > 3) = 1 - P(X \leq 3)$

$= 1 - 0.9978 = 0.0022$

table

(2)

A customer bought three boxes.

(c) Find the probability that only 2 of the boxes contained exactly 1 egg with a double yolk.

$X \sim B(3, 0.341)$

$P(X=2) = {}^{3}C_2 (0.341)^2 (0.659)^1$

$= 0.230$

(3)
(Total 8 marks)

6. Each cell of a certain animal contains 11000 genes. It is known that each gene has a probability 0.0005 of being damaged.

A cell is chosen at random.

(a) Suggest a suitable model for the distribution of the number of damaged genes in the cell.

Binomial

$$X \sim B(11000, 0.0005)$$

(2)

(b) Using a suitable approximation, find the probability that there are at most 2 damaged genes in the cell.

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= {}^{11000}C_0 (0.0005)^0 (0.9995)^{11000}$$

$$+ {}^{11000}C_1 (0.0005)^1 (0.9995)^{10999}$$

$$+ {}^{11000}C_2 (0.0005)^2 (0.9995)^{10998}$$

$$= 0.004081 + 0.022457576$$

$$+ 0.061783612 = 0.0883$$

(4)
(Total 8 marks)

7. A fair coin is tossed 4 times.

Find the probability that

(a) an equal number of heads and tails occur,

$$X \sim B(4, 0.5)$$

$$P(H=2) = {}^4C_2 (0.5)^2 (0.5)^2$$

$$= 0.375$$

(2)

(b) all the outcomes are the same,

P(HHHH) or P(TTTT)

$$0.5^4 + 0.5^4 = 0.125$$

(3)

(c) the first tail occurs on the third throw.

P(HHTT) + P(HHTH)

$$0.5^4 + 0.5^4 = 0.125$$

(2)
(Total 7 marks)

8. A manufacturer supplies DVD players to retailers in batches of 20. It has 5% of the players returned because they are faulty.

(a) Write down a suitable model for the distribution of the number of faulty DVD players in a batch.

Binomial $X \sim B(20, 0.05)$

(2)

Find the probability that a batch contains

(b) no faulty DVD players,

$$P(X=0) = {}^{20}C_0 (0.05)^0 (0.95)^{20}$$
$$= 0.35848$$

(2)

(c) more than 4 faulty DVD players.

$$P(X \geqslant 4) = 1 - P(X \leqslant 4)$$
$$= 1 - 0.9974$$ ← table
$$= 0.0026$$

(2)
(Total 6 marks)

9. A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.

A random sample of 10 patients with depression is taken from a doctor's records.

(a) Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

Binomial $X \sim B(10, 0.75)$

(2)

Given that the claim is correct,

(b) find the probability that the treatment will be successful for exactly 6 patients.

$$P(X=6) = {}^{10}C_6 (0.75)^6 (0.25)^4$$
$$= 0.145998$$
$$= 0.146 \text{ (3sf)}$$

(2)
(Total 4 marks)