Worksheet 8 Solutions

Question 1 Solution.

(a) The lines OA and OB lie along the x and y-axis respectively and so they are perpendicular and meet at right angles. Hence AB is the diameter of the circle passing through O, A and B.

(b) The circumcircle the triangle OAB is the circle passing through O, A and B.

The centre of the circle is the midpoint of A and B which is

$$\text{Midpoint of } AB = (4, -2)$$

The radius of the circle is half the distance between A and B:

$$\begin{aligned}\text{radius} &= \frac{1}{2}|AB| \\ &= \frac{1}{2}\sqrt{8^2 + (-4)^2} \\ &= \frac{1}{2}\sqrt{80} \\ &= 2\sqrt{5}\end{aligned}$$

So the equation of the circumcircle passing through O, A and B is

$$(x-4)^2 + (y+2)^2 = 20$$

Question 2 Solution.

(a) Since the equation has a root at $x = -0.4$, we must have that (since p and q are positive)

$$5(-0.4) + q = 0 \Rightarrow q = 5 \times 0.4 = 2$$

So $\boxed{q = 2}$.

(b) We need to differentiate and then set the derivative equal to 0 at $x = \frac{2}{15}\sqrt{3}$, since we know that the curve as a minimum at this point. This will give us an equation which we can use to find p. Before we can differentiate, we need to expand the brackets:

$$\begin{aligned} y &= x(px-2)(5x+2) \\ &= x(5px^2 + 2px - 10x - 4) \\ &= 5px^3 + 2px^2 - 10x^2 - 4x \end{aligned}$$

Differentiating this gives:

$$\begin{aligned} \frac{dy}{dx} &= 5(3)px^2 + 2(2)px - 20x - 4 \\ &= 15px^2 + 4px - 20x - 4 \end{aligned}$$

Now this is equal to 0 when $x = \frac{2}{15}\sqrt{3}$, so

$$\begin{aligned} &15p\left(\frac{2}{15}\sqrt{3}\right)^2 + 4p\left(\frac{2}{15}\sqrt{3}\right) - 20\left(\frac{2}{15}\sqrt{3}\right) - 4 = 0 \\ &\Rightarrow \frac{4}{5}p + \frac{8p}{15}\sqrt{3} = \frac{8}{3}\sqrt{3} + 4 \\ &\Rightarrow p\left(\frac{4}{5} + \frac{8}{15}\sqrt{3}\right) = \frac{8}{3}\sqrt{3} + 4 \\ &\Rightarrow p = \frac{\frac{8}{3}\sqrt{3} + 4}{\frac{4}{5} + \frac{8}{15}\sqrt{3}} = 5 \end{aligned}$$

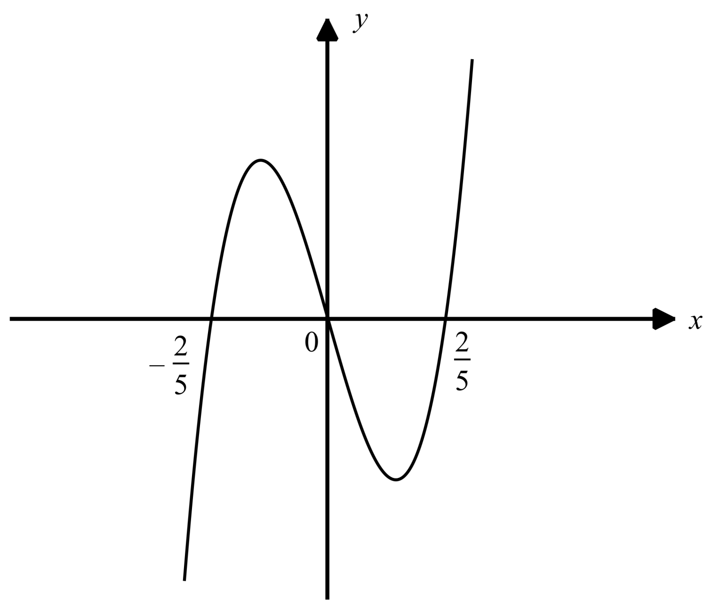
so $p = 5$ as required.

Exam Tip: When a question asks you to find the value of a constant or something similar, then you should always look for a way to construct an equation using the information given. Here we were given information about the roots of the equation and information about the minimum point. For part (b), we recalled that when a minimum point occurs, the derivative is 0; this knowledge helped us to form an equation, which is enough to find a single constant.

(c) Let's note some key features of the curve $y = x(5x+2)(5x-2)$ before we sketch it:

- This is a cubic curve and the x^3 term is positive.
- The curve crosses the y-axis when $x = 0$, so it crosses the y-axis at $y = 0(5(0)+2)(5(0)-2) = 0$.
- The curve crosses the x-axis when $y = 0$, so whenever $x(5x+2)(5x-2) = 0$, which is at $x = 0, x = \frac{5}{2}$ and $x = -\frac{5}{2}$.

Putting all this together, the sketch is:



Exam Tip: Even if you did not do parts (a) or (b), you can still do part (c): use the correct given value from part (b) and make some value up in the answer for (a), and draw your sketch with those values as you can still obtain any potential follow through marks. In general, you can still attempt other parts of the question with any given values you had to 'show' even if you were not able to show those values yourself. However, of course, if you do answer a question and get a value that's different from the one they've asked you to obtain, then use the correct value in the follow up questions.

Question 3 Solution.

(a)

$$6(1)^3 - 7(1)^2 + 1 = 6 - 7 + 1 = -1 + 1 = 0$$

so $x = 1$ solves the equation.

(b) If you use long division or inspection, you will find the solutions to the equation are $\boxed{x = 1, x = \frac{1}{2}, x = -\frac{1}{3}}$.

(c)

$$\begin{aligned}
&6\cos^3(\theta) - 1 = 5 - 7\sin^2(\theta) \\
&\Rightarrow 6\cos^3(\theta) - 1 = 5 - 7(1 - \cos^2(\theta)) \\
&\Rightarrow 6\cos^3(\theta) - 7\cos^2(\theta) + 1 = 0
\end{aligned}$$

This is a disguised version of the equation from parts (a) and (b), so the solutions are given when

$$\cos(\theta) = 1 \tag{1}$$

$$\cos(\theta) = \frac{1}{2} \tag{2}$$

$$\cos(\theta) = -\frac{1}{3} \tag{3}$$

(1) is simple: This is solved when $\theta = 0$ or $\theta = 360$.

(2) is also simple. It is solved when $\theta = 60$ or $\theta = 300$.

In the interval, $0 \leq \theta \leq 360$, (3) is solved when $\theta = 109.471...$ or $= 250.528...$

So overall, the solutions are $\boxed{\theta = 0,\ \theta = 60,\ \theta = 109.5,\ \theta = 250.5,\ \theta = 300 \text{ and } \theta = 360}$ to 1 decimal place.

Question 4 Solution.

(a) Considering the whole system and resolving upwards, we have

$$(\uparrow^{+}): \quad T - (0.8 + 0.2)g = (0.8 + 0.2)(3) \Rightarrow T = 12.8\text{N}$$

so the tension in the string is $\boxed{13 \text{ N}}$ to 2 significant figures.

(b) Consider block A, then

$$(\uparrow^{+}): \quad R - 0.2g = 0.2(3) \Rightarrow R = 2.56\text{N}$$

so the magnitude of the force exerted on A by B is $\boxed{2.6 \text{ N}}$ to 2 significant figures.

(c) By Newton's 3rd Law, the magnitude of the force exerted on B by A is $\boxed{2.6 \text{ N}}$ to 2 significant figures.

(d) We have used the fact that the lift is light by taking the mass of the lift to be 0.

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