1a)

$$y = \underset{u}{2x}\ \underset{v}{e^x}$$

$$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= \underline{\underline{2e^x + 2xe^x}}$$

b)

$$u = 3x^2 \qquad v = \ln 2x$$

$$\frac{du}{dx} = 6x \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$\underline{\underline{3x + 6x \ln 2x}}$$

2a)

$$u = x^2 \qquad v = (2x+1)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2$$

$$= (2x+1)^{-\frac{1}{2}}$$

$$\underline{\underline{2x(2x+1)^{\frac{1}{2}} + x^2(2x+1)^{-\frac{1}{2}}}}$$

b)

$$u = x \qquad v = \ln(x+1)$$

$$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{x+1}$$

$$\underline{\underline{\frac{x}{x+1} + \ln(x+1)}}$$

3a) $u = (x+5) \qquad v = (x+1)^3$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 3(x+1)^2$

$$\underline{\underline{(x+1)^3 + 3(x+1)^2 (x+5)}}$$

$$\left[\begin{array}{l} (x+1)^2 (x+1+3(x+5) \\ (x+1)^2 (4x+16) \\ 4(x+1)^2 (x+4) \end{array}\right] \text{(simplifying)}$$

b/ $u = x^3 \qquad v = \ln(2x+1)$

$\frac{du}{dx} = 3x^2 \qquad \frac{dv}{dx} = \frac{2}{2x+1}$

$$\underline{\underline{3x^2 \ln(2x+1) + \frac{2x^3}{2x+1}}}$$

4/ $y = (3x-1)\ln(2-x)$

$u = 3x-1 \qquad v = \ln(2-x)$

$\frac{du}{dx} = 3 \qquad \frac{dv}{dx} = \frac{-1}{2-x}$

$$\frac{dy}{dx} = 3\ln(2-x) - \left(\frac{3x-1}{2-x}\right)$$

when $x = 1$ $\quad \frac{dy}{dx} = 3\ln 1 - 2$

$= -2$

$y = 2\ln 1$

$= 0$

$y = -2x + c \qquad (1, 0)$

$0 = -2 + c$

$c = 2$

$$\underline{\underline{y = -2x + 2}}$$

5) $y = x(x-3)^3$

$u = x \qquad v = (x-3)^3$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 3(x-3)^2$

$\frac{dy}{dx} = 3x(x-3)^2 + (x-3)^3$

$= (x-3)^2(3x + x - 3)$

$= (x-3)^2(4x-3)$

~~tur~~. stationary points where $\frac{dy}{dx} = 0$

$(x-3)^2(4x-3) = 0$

$x = 3 \qquad x = \frac{3}{4}$

$y = 3(0)^3 \qquad y = \frac{3}{4}\left(\frac{3}{4} - 3\right)^3$

$= 0$

$= \frac{-2187}{256}$

$(3, 0) \qquad \left(\frac{3}{4}, \frac{-2187}{256}\right)$

$\frac{dy}{dx} = (x-3)^2(4x-3)$

$u = (x-3)^2 \qquad v = (4x-3)$

$\frac{du}{dx} = 2(x-3) \qquad \frac{dv}{dx} = 4$

$\frac{d^2y}{dx^2} = 4(x-3)^2 + 2(x-3)(4x-3)$

when $x = 3$ $\frac{d^2y}{dx^2} = 0$

when $x = 2.9$ $\frac{dy}{dx} = 0.086$

when $x = 3.1$ $\frac{dy}{dx} = 0.094$

$(3, 0)$: POINT OF INFLEXION

when $x = \frac{3}{4}$ $\frac{d^2y}{dx^2} = \frac{81}{4}$

$\left(\frac{3}{4}, \frac{-2187}{256}\right)$ MINIMUM

6 $y = x(x-1)^{\frac{1}{2}}$

$u = x \qquad v = (x-1)^{\frac{1}{2}}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2}(x-1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x(x-1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}}$

when $x = 5$

$\frac{dy}{dx} = \frac{1}{2}(5)(4)^{-\frac{1}{2}} + (4)^{\frac{1}{2}}$

$= \frac{13}{4}$

$y = 5(4)^{\frac{1}{2}}$

$= 10$

normal gradient $= -\frac{4}{13}$

$y = -\frac{4}{13}x + c \qquad (5, 10)$

$10 = -\frac{20}{13} + c$

$c = \frac{150}{13}$

$y = \frac{-4}{13}x + \frac{150}{13}$

7/

$$y = x e^{x^2}$$

$$u = x \qquad v = e^{x^2}$$

$$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 2x e^{x^2}$$

$$\frac{dy}{dx} = e^{x^2} + 2x^2 e^{x^2}$$

when $x = 1$ $\quad \frac{dy}{dx} = e + 2e$

$$= 3e$$

$$y = 1 e^1$$
$$= e$$

$$y = 3e x + c \qquad (1, e)$$
$$e = 3e + c$$
$$c = -2e$$

$$\underline{\underline{y = 3e x - 2e}}$$

b/ $B: (0, -2e)$

Crosses x when $y = 0$

$$0 = 3e x - 2e$$
$$2e = 3e x$$
$$x = \frac{2}{3}$$

$$\text{Area} = \frac{1}{2} \times 2e \times \frac{2}{3}$$

$$= \underline{\underline{\frac{2}{3} e \text{ units}^2}}$$