AS-Level Mathematics - Pure Maths Test - ‘INTERMEDIATE’ Solutions

1. (a) **Factorise** $9x^2 - 4y^2$

   (b) **Simplify** $(4x^2)^{\frac{-3}{2}}$

   (c) **Rationalise the denominator of** $\frac{\sqrt{5}-2}{\sqrt{5}+3}$

   (a) In order to factorise a difference of squares, we must identify what is being squared in each of the terms. Once identified, put into brackets, one with a plus and one with a minus:

$$(3x + 2y)(3x - 2y)$$

   Expanding out these brackets, you will see which terms cancel to leave $9x^2 - 4y^2$.

   (b)

$$\begin{aligned}\left(4x^2\right)^{\frac{-3}{2}} &= 4^{\frac{-3}{2}}\left(x^2\right)^{\frac{-3}{2}} \\ &= \left(4^{\frac{1}{2}}\right)^{-3}\left(\left(x^2\right)^{\frac{1}{2}}\right)^{-3} \\ &= 2^{-3}x^{-3} \\ &= \frac{1}{8x^3}\end{aligned}$$

   (c)

$$\begin{aligned}\frac{\sqrt{5}-2}{\sqrt{5}+3} &= \frac{\sqrt{5}-2}{\sqrt{5}+3} \times \frac{\sqrt{5}-3}{\sqrt{5}-3} \\ &= \frac{\left(\sqrt{5}-2\right)\left(\sqrt{5}-3\right)}{\left(\sqrt{5}+3\right)\left(\sqrt{5}-3\right)} \\ &= \frac{5-2\sqrt{5}-3\sqrt{5}+6}{5+3\sqrt{5}-3\sqrt{5}-9} \\ &= \frac{11-5\sqrt{5}}{-4} \\ &= \frac{5\sqrt{5}-11}{4}\end{aligned}$$

2. (a) **Show that $x - 2$ is a factor of $f(x) = 2x^3 - 3x^2 - 5x + 6$.**

The Factor Theorem states that if $f(a) = 0$, then $x - a$ is a factor of $f(x)$. So, in order to show that $x - 2$ is a factor of $f(x)$ we must show that $f(2) = 0$:

$$\begin{aligned} f(2) &= 2 \times 2^3 - 3 \times 2^2 - 5 \times 2 + 6 \\ &= 2 \times 8 - 3 \times 4 - 5 \times 2 + 6 \\ &= 16 - 12 - 10 + 6 \\ &= 0 \end{aligned}$$

(b) **Factorise $f(x)$ completely.**

From part a) we know that $x - 2$ is a factor. You can use polynomial division to divide $f(x)$ by $x - 2$ and see what the other factor(s) are or you can use the following method, use whatever you prefer. $x - 2$ is a factor and so $f(x)$ can be written as:

$$f(x) = (x - 2)(ax^2 + bx + c)$$

(We know that the other factor must be a quadratic because $f(x)$ is a cubic). Multiply this out:

$$\begin{aligned} f(x) &= (x - 2)(ax^2 + bx + c) \\ &= ax^3 - 2ax^2 + bx^2 - 2bx + cx - 2c \\ &= ax^3 + (b - 2a)x^2 + (c - 2b)x - 2c \qquad \text{(by collecting like terms)} \end{aligned}$$

Therefore, $ax^3 + (b - 2a)x^2 + (c - 2b)x - 2c$ is the same as $2x^3 - 3x^2 - 5x + 6$. By equating the coefficients of the terms we get:

$$\begin{aligned} &a = 2, \\ &b - 2a = b - 4 = -3, \quad \Rightarrow b = 1, \\ &c - 2b = c - 2 = -5, \quad \Rightarrow c = -3, \end{aligned}$$

and $c = -3$ agrees with $-2c = 6$. Hence, we have

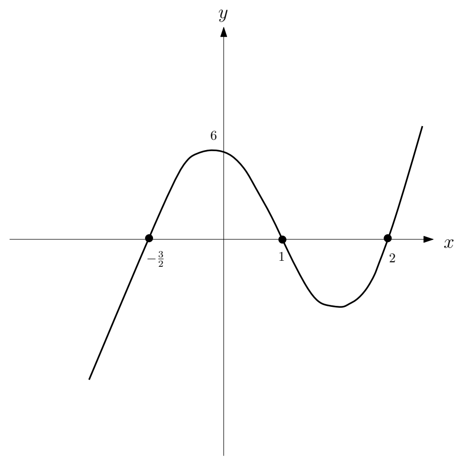
$$f(x) = (x - 2)\left(2x^2 + x - 3\right)$$

Don't forget that the quadratic factor can also be factorised and so $f(x)$, factorised fully, is given by

$$f(x) = (x - 2)(2x + 3)(x - 1).$$

(c) **Sketch $f(x)$.**

When $f(x)$ is factorised, it is easy to see where its roots are (the roots are where $f(x)$ crosses the x-axis, i.e. when $f(x) = 0$). $f(x)$ is 0 when $x = 2$, $x = -\frac{3}{2}$ or $x = 1$. The overall shape is that of a positive cubic and so the sketch is given by



Note that if you are required to label all points where the graph crosses an axis, you can also find where it crosses the y-axis by putting $x = 0$ into $f(x)$ (you should get 6).

3. **(a) Determine the set of values of k for which the equation $x^2 + 2x - k = 0$ has 2 real solutions.**

(b) Suppose $k = 2$. By first completing the square, sketch the graph of $y = x^2 + 2x - k$, labelling clearly any intersection of axes and the vertex of the graph.

(a) A quadratic equation of the form $ax^2 + bx + c = 0$ has 2 real solutions when the discriminant, $b^2 - 4ac$, is greater than 0. In this case, $a = 1$, $b = 2$ and $c = -k$. Hence, this quadratic has 2 real solutions when

$$\begin{aligned} 2^2 - 4 \times 1 \times -k &> 0 \\ \Rightarrow \quad 4 + 4k &> 0 \\ \Rightarrow \quad 4k &> -4 \\ \Rightarrow \quad k &> -1 \end{aligned}$$

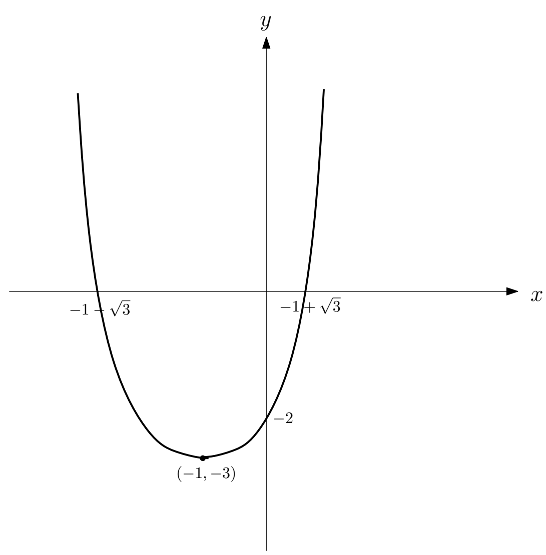
(b) If $k = 2$, then since this is more than -1, from part a) the equation has 2 real solutions. These solutions occur when

$$\begin{aligned} x^2+2x-2 &= 0 \\ \Rightarrow \quad (x+1)^2-1-2 &= 0 \\ \Rightarrow \quad (x+1)^2-3 &= 0 \end{aligned}$$

Note that the graph of $(x+1)^2-3$ is the graph of x^2 shifted to the left by 1 then shifted downwards by 3: $x^2 \to (x+1)^2 \to (x+1)^2-3$. The roots of the graph are given by the solutions of

$$\begin{aligned} (x+1)^2-3 &= 0 \\ \Rightarrow \quad (x+1)^2 &= 3 \\ \Rightarrow \quad x+1 &= \pm\sqrt{3} \\ \Rightarrow \quad x &= -1\pm\sqrt{3} \end{aligned}$$

The graph also intersects the y-axis at $y=-2$ and the vertex is at (-1,-3) as seen from the transformations.



4. **Solve the following simultaneous equations:**

$$\begin{aligned} 2y+x-3 &= 0 \\ x^2+3xy-10 &= 0 \end{aligned}$$

The first equation gives us $x=3-2y$. Substituting this into the second equation we obtain:

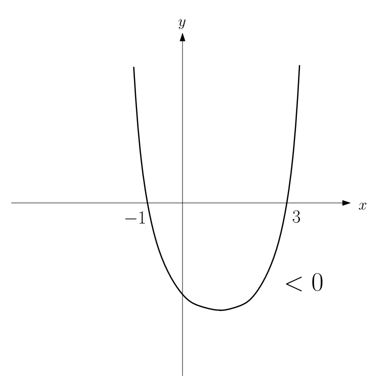
$$\begin{aligned} (3-2y)^2+3(3-2y)y-10 &= 0 \\ \Rightarrow \quad (3-2y)(3-2y)+(9-6y)y-10 &= 0 \\ \Rightarrow \quad (9-6y-6y+4y^2)+(9y-6y^2)-10 &= 0 \\ \Rightarrow \quad -2y^2-3y-1 &= 0 \\ \Rightarrow \quad 2y^2+3y+1 &= 0 \\ \Rightarrow \quad (2y+1)(y+1) &= 0 \\ \Rightarrow \quad 2y+1=0 \quad \text{OR} \quad & y+1=0 \\ \Rightarrow \quad y=-\frac{1}{2} \quad \text{OR} \quad & y=-1 \end{aligned}$$

The corresponding x values, found using the first equation, are given by $3-2\left(-\frac{1}{2}\right)=4$ and $3-2(-1)=5$. Hence, the solutions are $(x_1,y_1)=\left(4,-\frac{1}{2}\right)$ and $(x_2,y_2)=(5,-1)$.

5. **Find the values of x for which $x^2<2x+3$.**

$$\begin{aligned} x^2 &< 2x+3 \\ x^2-2x-3 &< 0 \\ (x-3)(x+1) &< 0 \end{aligned}$$

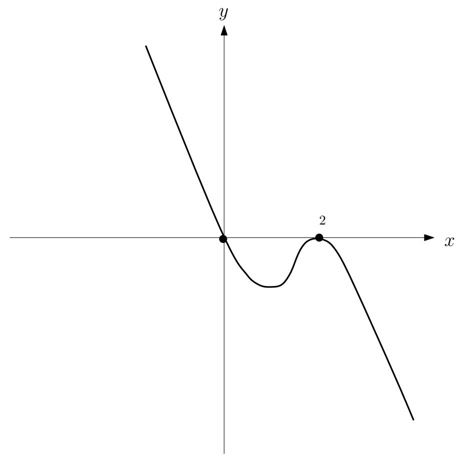
By sketching the curve of $f(x)=(x-3)(x+1)$ we can see where it is less than 0:



The graph is less than 0 over the interval $-1<x<3$, and so this interval solves the inequality.

6. **Sketch the graph of $f(x)=4x^2-4x-x^3$.**

First factorise to find the roots of the function: $f(x)=-x(x^2-4x+4)=-x(x-2)^2$. There is a root at $x=0$ and a repeated root at $x=2$, i.e. the graph cuts the x-axis at $x = 0$ and touches the x-axis at $x = 2$. The overall shape is that of a negative cubic and so the graph is given by

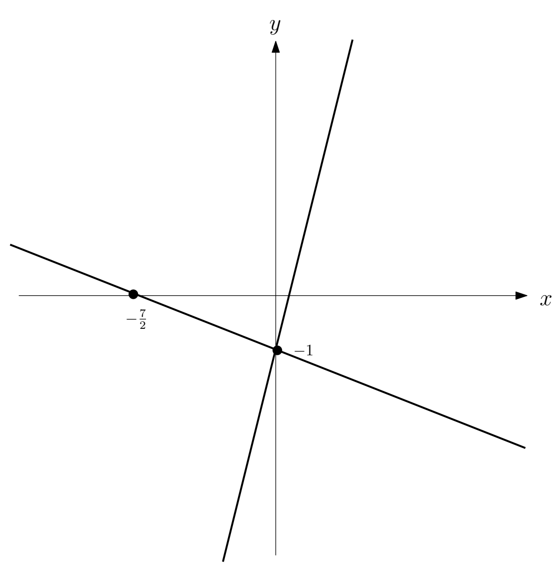


7. **The line l_1 passes through the points $A(2, 6)$ and $B(0, -1)$. The line l_2 is perpendicular to l_1 and intersects l_1 at the point B.**

   (a) **Find the equation of the line l_1 in the form $ax + by + c = 0$ where a, b and c are integers.**

   (b) **Find the coordinates of the point where l_2 intersects the x-axis.**

   (a) Always sketch the information given to you, this allows to see whether the answers you obtain seem feasible and may sometimes even show you more information than expected.



First find the equation of l_1 in the form $y = mx + c$. The gradient is given by:

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - (-1)}{2 - 0} = \frac{7}{2}$$

l_1 intercepts the y-axis at -1, so the equation of l_1 is given by

$$y = \frac{7}{2}x - 1$$

To get the equation in the form $ax + by + c = 0$:

$$\begin{aligned} y &= \frac{7}{2}x - 1 \\ \Rightarrow \quad 2y &= 7x - 2 \\ \Rightarrow \quad 7x - 2y - 2 &= 0 \end{aligned}$$

Hence, $a = 7$, $b = -2$ and $c = -2$.

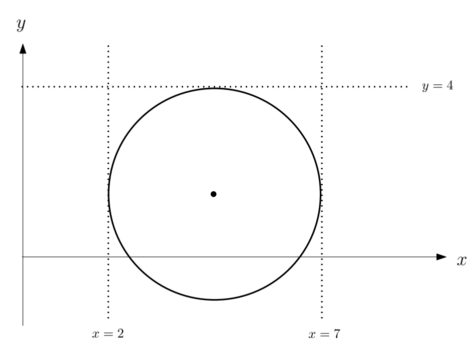
(b) Since l_2 is perpendicular to l_1, the gradient of l_2 is given by $-\frac{2}{7}$ and the equation of l_2 takes the form $y = -\frac{2}{7}x + c$. l_2 intersects the y-axis at -1 and so the equation of l_2 is given by $y = -\frac{2}{7}x - 1$. Setting y equal to zero allows us to find the x-coordinate of where l_2 crosses the x-axis:

$$\begin{aligned} 0 &= -\frac{2}{7}x - 1 \\ \Rightarrow \quad \frac{2}{7}x &= -1 \\ \Rightarrow \quad x &= -\frac{7}{2} \end{aligned}$$

Hence, the coordinates of where l_2 crosses the x-axis are given by $\left(\frac{-7}{2}, 0\right)$.

8. **The lines $x = 2$ and $x = 7$ are tangent to a circle and $y = 4$ touches the top of the circle. Find the equation of the circle in the form $(ax + b)^2 + (ay + c)^2 = d$, where a, b, c and d are integers.**

First draw the circle as described by the tangent lines.



The equation of a circle is given by $(x-a)^2+(y-b)^2=r^2$, where (a,b) is the centre of the circle and r is the radius. a, b and r must be specified in this equation. Since the centre of the circle lies half way between the tangent $x=2$ and $x=7$, the radius of the circle is $\frac{5}{2}$ and the x-coordinate of the centre is $x=\frac{9}{2}$. Similarly, the y-coordinate of the centre is $\frac{3}{2}$. We have the equation of the circle:

$$\left(x-\frac{9}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{5}{2}\right)^2$$

The question asks us to rewrite the equation in a specific format; all the coefficients must be integers. Consider the r^2 in the formula. $r^2=\left(\frac{5}{2}\right)^2=\frac{25}{4}$ and so if we multiply both sides by 4 we will at least have an integer on the right hand side:

$$\begin{aligned}
\left(x-\frac{9}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 &= \frac{25}{4}\\
\implies\quad 4\left(x-\frac{9}{2}\right)^2+4\left(y-\frac{3}{2}\right)^2 &= 25\\
\implies\quad 2^2\left(x-\frac{9}{2}\right)^2+2^2\left(y-\frac{3}{2}\right)^2 &= 25\\
\implies\quad \left(2\left(x-\frac{9}{2}\right)\right)^2+\left(2\left(y-\frac{3}{2}\right)\right)^2 &= 25\\
\implies\quad (2x-9)^2+(2y-3)^2 &= 25
\end{aligned}$$

By writing 4 as 2^2, we can take the 2 inside the squaring process (multiply $(2x+9)^2$ out to see if you agree). Now all the coefficients are integers as requested by the question: $a=2$, $b=-9$, $c=-3$, $d=25$.

9. **If x is so small that x to the power of 3 or higher can be ignored, show that**

$$(3-x)(1+2x)^4\approx 3+23x+64x^2.$$

In order to see $(3-x)(1+2x)^4$ in terms of powers of x, we must expand $(1+2x)^4$ in powers of x using the formula for binomial expansion as given in the formula booklet:

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+...$$

where

$$\binom{n}{r}={}^nC_r$$

There is an ${}^nC_r$ (n chose r) button on your calculator. If you make a direct substitution into the formula with the following values:

$$a \to 1$$
$$b \to 2x$$
$$n \to 4$$

then the formula becomes:

$$\begin{aligned}(1+2x)^4 &= 1^4 + \begin{pmatrix} 4 \\ 1 \end{pmatrix} 1^{4-1}(2x) + \begin{pmatrix} 4 \\ 2 \end{pmatrix} 1^{4-2}(2x)^2 + ... \\ &= 1 + 4 \times 1 \times 2x + 6 \times 1 \times 4x^2 + ... \\ &= 1 + 8x + 24x^2 + ...\end{aligned}$$

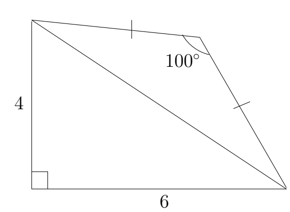
The original expression becomes

$$\begin{aligned}(3-x)(1+2x)^4 &= (3-x)\left(1 + 8x + 24x^2 + ...\right) \\ &= 3 - x + 24x - 8x^2 + 72x^2 - 24x^3 + ... \\ &= 3 + 23x + 64x^2 - 24x^3 + ...\end{aligned}$$

Since we are told that x to the power of 3 or more can be ignored the final equation is given by

$$(3-x)(1+2x)^4 = 3 + 23x + 64x^2.$$

10. **Calculate the area of this quadrilateral:**



The area of this quadrilateral can be found by finding the area of the two triangles of which it is comprised. The area of the first triangle, the right-angled one, can be found using the formula

$$\text{Area of Triangle } = \frac{1}{2} \times \text{ base } \times \text{ height}$$

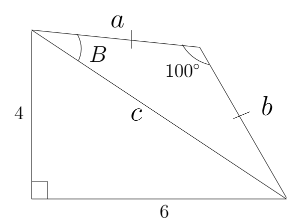
We use this formula because we know the base and the height. The area of this triangle is

$$\frac{1}{2} \times 4 \times 6 = 12$$

The area of the second triangle can be found using the formula

$$\text{Area of Triangle } = \frac{1}{2}ab\sin(C)$$

where if a and b are 2 sides then C is the angle between them. We use this formula because we do not have the height of the triangle. Consider the quadrilateral with the given labels as follows:



To find sides a and b we must first find the longest side, c. This can be found by applying pythagoras to the right-angled triangle:

$$\begin{aligned} 4^2 + 6^2 &= c^2 \\ \Rightarrow 16 + 36 = 52 &= c^2 \\ \Rightarrow c &= \sqrt{52} \approx 7.211 \end{aligned}$$

Recall the sine rule for finding missing sides and angles:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Angle B is given by $40°$ and so

$$\frac{b}{\sin(40)} = \frac{7.211}{\sin(100)} \quad \Longrightarrow \quad b = \frac{7.211}{\sin(100)} \times \sin(40) \approx 4.707$$

Since the triangle is isosceles a is also 4.707. Hence, the area of the second triangle is

$$\frac{1}{2} \times 4.707 \times 4.707 \times \sin(100) \approx 10.908$$

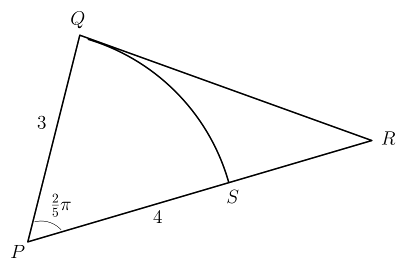
The combined area, i.e. the area of the quadrilateral is given by

$$\text{Area of Quadrilateral } \approx 12 + 10.908 = 22.908\text{units}^2$$

Be careful to carry the accuracy through the question especially when using trig.

11. **Triangle PQR is such that $PQ = 3$cm, $PR = 4$cm and angle $QPR = \frac{2}{5}\pi$. An arc of a circle, centre at P and radius 3cm starts at Q and cuts PR at S. Find the perimeter and area of the region enclosed by the arc QS and the sides SR and QR.**

Begin by drawing (not accurately) the shape described.



The length SR is 1 and so we require the lengths QR and QS to find the perimeter of the region QRS. Length QR can be found using the cosine rule:

$$\begin{aligned} \text{QR}^2 &= \text{PQ}^2 + \text{PR}^2 - 2 \times \text{PQ} \times \text{PR} \cos(\text{QPR}) \\ \Rightarrow \quad \text{QR}^2 &= 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos\left(\frac{2}{5}\pi\right) \\ \Rightarrow \quad \text{QR}^2 &= 25 - 24\cos\left(\frac{2}{5}\pi\right) \\ \Rightarrow \quad \text{QR}^2 &= 1.0058... \\ \Rightarrow \quad \text{QR} &\approx 1.0029 \end{aligned}$$

The length QS can be found using the formula for the length of an arc $r\theta$ where θ is measured in radians, i.e. $\text{QS} = 3 \times \frac{2}{5}\pi \approx 3.7699$. It follows that the perimeter of the shape is given by

$$\text{Perimeter} \approx 1 + 1.0029 + 3.7699 = 5.7728$$

to 4 decimal places.

The area of the region can be found by subtracting the area of the sector from the area of the triangle. The area of the sector can be found using the formula $\frac{1}{2}r^2\theta$ where θ is measure in radians, i.e. $\frac{1}{2} \times 3^2 \times \frac{2}{5}\pi \approx 5.6549$. The area of the triangle can be found using the formula $\frac{1}{2}ab\sin(C)$, i.e. $\frac{1}{2} \times 3 \times 4 \times \sin\left(\frac{2}{5}\pi\right) \approx 5.7063$. It follows that the area of the region is given by

$$\text{Area} \approx 5.7063 - 5.6549 = 0.0514$$

12. **Without the use of a calculator, evaluate the following:**

    (a) $\cos(270) = 0$.

    (b) $\sin\left(\frac{-\pi}{2}\right) = -1$.

    (c) $\tan(180) = 0$.

13. **Solve** $\cos(4t) = \frac{2}{3}$ **on the interval** $0 \leq t \leq 360$**.**

    If $0 \leq t \leq 360$ then $0 \leq 4t \leq 1440$. If we let $\theta = 4t$ then we need to solve the equation $\cos(\theta) = \frac{2}{3}$ on the interval $0 \leq \theta \leq 1440$ for θ. $\cos^{-1}\left(\frac{2}{3}\right)$ gives the first θ value as 48.19 to 2 decimal places. Using the symmetry of the cosine graph, the second θ value is $360 - 48.19 = 311.81$. These two solutions will repeat every 360° due to the periodicity of the cosine graph giving the full set of θ solutions in the range $0 \leq \theta \leq 1440$ as $48.19°, 311.81°, 408.19°, 671.81°, 768.19°, 1031.81°, 1128.19°$ and $1391.81°$. Since $\theta = 4t$, we can find the full set of t solutions by dividing the θ solutions by 4:

    $$t = 12.05°, 77.95°, 102.05°, 167.95°, 192.05°, 257.95°282.05°, 347.95°.$$

14. **Solve** $2\log_4(x) - \log_4(2x - 3) = 1$ **for** x**.**

    In order to solve this logarithmic equation we must manipulate the left-hand side into a single logarithm using any or all the following rules:

    $$\log_a(b) + \log_a(c) = \log_a(bc) \tag{1}$$

    $$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right) \tag{2}$$

    $$n\log_a(b) = \log_a(b^n) \tag{3}$$

    Rule (3) allows us to write the equation as

    $$\log_4(x^2) - \log_4(2x - 3) = 1$$

    Rule (2) then allows us to write this as

    $$\log_4\left(\frac{x^2}{2x - 3}\right) = 1$$

    This can be read as 'the power of 4 that gives $\frac{x^2}{2x-3}$ is 1', i.e.

    $$4^1 = \frac{x^2}{2x - 3}$$

This can now be turned into a simple quadratic by multiplying both sides by $(2x-3)$:

$$\begin{aligned} 4(2x-3) &= x^2 \\ \Rightarrow \quad x^2 - 8x + 12 &= 0 \\ \Rightarrow \quad (x-2)(x-6) &= 0 \end{aligned}$$

Hence, the solutions are $x = 6$ and $x = 2$.

15. **Find y given that $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$ and y passes through the point $(9, 9)$.**

$\frac{dy}{dx}$ can be written as $\frac{dy}{dx} = x^{-\frac{1}{2}}$ and can be integrated to give $y = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = 2x^{\frac{1}{2}} + c$. Substituting the coordinates (9,9) gives $9 = 2(9)^{\frac{1}{2}} + c = 6 + c$ and so $c = 3$. It follows that $y = 2x^{\frac{1}{2}} + 3$.

16. **Find and classify the stationary points of $y = 2x^3 - 3x^2 - 36x + 14$.**

The stationary points of a curve are the maximum or minimum points. They are so called because at those points they are neither going up or down; the curve has zero gradient at these points. By differentiating y, we can find the function, $\frac{dy}{dx}$, that tells us the gradient at any x-coordinate:

$$\frac{dy}{dx} = 6x^2 - 6x - 36.$$

Note that constants disappear when you differentiate because there are no powers of x; this is the same as saying flat lines, e.g. $y = 14$, have zero gradient.
We can find all the points that have a zero gradient by specifying that $\frac{dy}{dx}$ be equal to zero and solving for the x-coordinates:

$$\begin{aligned} \frac{dy}{dx} &= 0 \\ \Longrightarrow \quad 6x^2 - 6x - 36 &= 0 \\ \Longrightarrow \quad x^2 - x - 6 &= 0 \\ \Longrightarrow \quad (x+2)(x-3) &= 0 \end{aligned}$$

This gives us the x coordinates of the stationary points: $x = -2$, $x = 3$. The y coordinates to go with these x coordinates are given by

$$y = 2(-2)^3 - 3(-2)^2 - 36 \times -2 + 14 = 2 \times -8 - 3 \times 4 + 72 + 14 = 58,$$
$$y = 2(3)^3 - 3(3)^2 - 36 \times 3 + 14 = 2 \times 27 - 3 \times 9 + 108 + 14 = 149$$

Hence, the stationary points are: $(-2, 58)$, $(3, 149)$.
We classify stationary points by putting the x-coordinates into the second derivative.
The second derivative is found by differentiating y twice (or differentiate $\frac{dy}{dx}$ once):

$$\frac{d^2y}{dx^2} = 12x - 6$$

If $\frac{d^2y}{dx^2}$ is greater than 0 at the chosen x-coordinate, that stationary point is a MINIMUM.
If $\frac{d^2y}{dx^2}$ is less than 0 at the chosen x-coordinate, that stationary point ia a MAXIMUM.

$$\frac{d^2y}{dx^2} = 12 \times -2 - 6 = -30 < 0,$$
$$\frac{d^2y}{dx^2} = 12 \times 3 - 6 = 30 > 0.$$

Hence, $(-2, 58)$ is a MAXIMUM and $(3, 149)$ is a MINIMUM.

17. **Find the equation of the tangent to the curve of $y = x^2(1 - x)$ at the point where $x = 1$.**

In order to find the equation of a tangent at a point, we must know the point and the gradient at that point. Since the x coordinate is 1, the y coordinate is given by $y = 1^2(1 - 1) = 0$. The gradient can be found by first expanding: $y = x^2 - x^3$ and then differentiating to find a formula for the gradient:

$$\frac{dy}{dx} = 2x - 3x^2$$

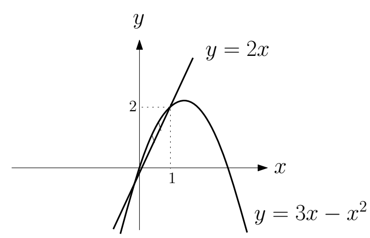
At the point $(1, 0)$, the gradient is $2(1) - 3(1)^2 = 2 - 3 = -1$. The tangent thus takes the form $y = -x + c$. We find c by substituting in the coordinates of our point: $0 = -1 \times 1 + c$, hence $c = 1$. The equation of the tangent is $y = -x + 1$.

18. (a) **Find the x-coordinates of where the graphs of $y = 3x - x^2$ and $y = 2x$ intersect.**

(b) **Hence, find the area of the region enclosed by the two graphs.**

(a) Finding the coordinates of the points where two curves intersect is the same as solving the equations simultaneously, i.e. solving $3x - x^2 = 2x$ by substituting for y. Simplifying and factorising gives $x(1 - x) = 0$ and so $x = 0$ or $x = 1$.

(b) Sketch the curves and the area to find:



The area enclosed between the two curves can be found by first finding the area beneath the curve $y = 3x - x^2$ between $x = 0$ and $x = 1$. This can be done using integration:

$$\int_0^1 \left(3x - x^2\right) dx = \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \left(\frac{3}{2} - \frac{1}{3}\right) - 0 = \frac{9}{6} - \frac{2}{6} = \frac{7}{6}.$$

The space enclosed between the straight line and the x-axis is a triangle with base 1 and height 2 and so has area 1. The area enclosed is then $\frac{7}{2} - 1 = \frac{5}{2}$.