In questions that tell you to show your working, you shouldn't depend solely on a calculator. For these questions, solutions based entirely on graphical or numerical methods are not acceptable.

1 $y = 3x^2 - \dfrac{2}{\sqrt{x}}$

a Find $\dfrac{dy}{dx}$ **[2 marks]**

b Calculate the rate of change of y with respect to x at the point where $x = 4$ **[2]**

c Find the equation of the normal to the curve at the point where $x = 1$ **[5]**

2 The volume (in m^3) of water in a tank at time t seconds is given by

$$V = t - \frac{4}{t^2}$$

At what time will the rate of change of the volume be $2\,m^3 s^{-1}$? Show your working. **[4]**

3 **a** Differentiate the following expression with respect to x

i $5\cos x$ **ii** $\ln x$ **iii** $\dfrac{1}{4}e^x$ **[3]**

b Find the equation of the normal to $y = 2\ln x$ at the point where $x = 1$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. **[5]**

4 **a** Given that $y = xe^x$ find expressions for **i** $\dfrac{dy}{dx}$ **ii** $\dfrac{d^2y}{dx^2}$ **[4]**

b Write down an expression for the k^{th} derivative of $y = xe^x$ **[1]**

5 $f(x) = 3x\sin x$

a Calculate $f'(x)$ **[2]**

b Find an equation of the tangent to $y = f(x)$ at the point where $x = \pi$ **[3]**

6 **a** Differentiate these expressions with respect to x

i $\dfrac{x}{x+2}$ **ii** $\dfrac{3x^2}{\cos x}$ **iii** $(3x^3 + 5)e^x$ **[6]**

b Show that the derivative of $\dfrac{x^2 + 3x}{x - 5}$ can be written as $\dfrac{ax^2 + bx + c}{(x-5)^2}$, where a, b and c are constants to be found. **[3]**

7 **a** Differentiate these expressions with respect to x

i $\cos 3x$ **ii** $2e^{3x}$ **iii** $\sin(2x - 5)$ **[3]**

b Calculate the exact gradient of the curve of $y = \sin^2 x$ at the point where $x = \dfrac{\pi}{3}$.
Show your working. **[3]**

8 Given that $x = 2y^2 - 8\sqrt{y}$

a Find **i** $\frac{dx}{dy}$ **ii** $\frac{dy}{dx}$ in terms of y **[4]**

b Work out the equation of the normal at the point where $y = 4$ **[4]**

9 Find $\frac{dy}{dx}$ given that $5xy - y^3 = 7$ **[4]**

10 A curve is defined by the parametric equations $x = t^2$, $y = 6t$, $t > 0$

a Calculate the gradient of the curve when $y = 18$. Show your working. **[4]**

b Find the equation of the tangent at the point where $x = 4$ **[4]**

11 $y = 2x^2 e^x$

a Use calculus to find the stationary points of the curve. Show your working. **[6]**

b Classify each stationary point. **[4]**

12 A curve C has the equation $y = x^3 - 4x + 3$

a Identify and describe any points of inflection on the curve. **[4]**

b Sketch the curve C **[2]**

c For what values of x is the curve concave? **[1]**

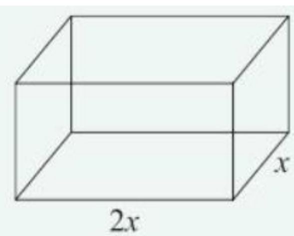
13 A curve C has the equation $y = x^2e^x + e^x$

a Showing your working, find the stationary point on the curve and show that it is a point of inflection. **[5]**

b By considering $\frac{d^2y}{dx^2}$, show that the curve has another point of inflection. **[4]**

14 A cuboid has length twice its width as shown.

The volume of the cuboid is $192\,cm^3$



a Show that the surface area of the cuboid, S, is given by $S = 4x^2 + \frac{k}{x}$, where k is a constant to be found. **[5]**

b Find the minimum value of S, showing your working. **[5]**

c Use calculus to justify this is a minimum. **[3]**

15 Find and classify the stationary point on the curve $y = x^3 + 3x^2 + 3x - 5$. Show your working. **[5]**

16 Use differentiation from first principles to find the derivative of

a $\sin x$ **[4]** **b** $\cos 2x$ **[4]**

17 $f(x) = \tan x$

a Write down $f'(x)$ **[1]**

b Show that $f''(x) = 2\tan x \sec^2 x$ **[3]**

18 You are given that $y = 4^x$

a Write down $\frac{dy}{dx}$ **[1]**

b The tangent to the curve at the point $(-1, 0.25)$ cuts the x-axis at point A

Find the exact x-coordinate of A **[4]**

19 Given that $y = e^x \sin 5x$

a Calculate **i** $\frac{dy}{dx}$ **ii** $\frac{d^2y}{dx^2}$ **[6]**

b Find the smallest positive value of x to give a maximum value of y and prove it is a maximum. Show your working. **[5]**

20 a Differentiate the following with respect to t

i $\frac{e^{3t}}{t^2+1}$ **ii** $3t\ln t$ **iii** $e^{-t}\sin 4t$ **[8]**

b Find $g'(x)$, when $g(x) = 2^x \tan x$ **[2]**

21 Given that $y = \sec x$

a Prove that $\frac{dy}{dx} = \sec x \tan x$ **[4]**

b Find $\frac{d^2y}{dx^2}$, giving your answer in terms of $\sec x$ only. **[3]**

22 The value of money in an account after t years is approximated by the formula

$V = ke^{0.03t}$, $k > 0$ is a constant.

Given that £5000 is invested originally,

a Work out the value of the account after 10 years, **[3]**

b Calculate the rate in increase in value when $t = 5$. Show your working. **[3]**

23 A curve C has equation $x = y\ln y$

a Calculate $\frac{dy}{dx}$ at the point where $y = 2$. Show your working. **[4]**

The normal to C at the point where $y = 2$ intersects the y-axis at point A

b Find the exact y-coordinate of A. Show your working. **[5]**

24 Given that $x = \cos y$, show that $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$ **[4]**

25 Find an expression for the gradient in each case.

a $xe^y - \ln x = 12$ **[4]**

b $x\sin y - 2x^2 y = 0$ **[4]**

26 A curve C has equation $\sin x + \frac{1}{2}\tan y = \sqrt{3}$, $0 \le x, y \le \pi$

Find $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$ **[7]**

27 A curve has parametric equations $x = 3t + 7$, $y = 2 + \frac{3}{t}$, $t \neq 0$

a Find the gradient of the curve at the point where $x = 10$. Show your working. **[4]**

b Find a Cartesian equation of the curve in the form $y = f(x)$ where $f(x)$ is expressed as a single fraction. **[3]**

28 A curve is defined by the parametric equations $x = 2\sin t$, $y = 3\cos t$

a Find an expression for $\frac{dy}{dx}$ **[3]**

b Work out the equation of the tangent at the point when $t = \frac{2\pi}{3}$

Give your answer in the form $a\sqrt{3}x + by + c = 0$, where a, b and c are integers. **[4]**

29 Find and classify the stationary point of the curve $y = x^4 - 3$. Show your working. **[5]**

30 Use calculus to determine if the following functions are convex or concave.

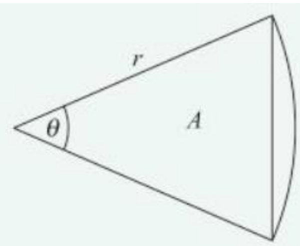
a $f(x) = 3x^2 - 7x + 8$ **[3]**

b $g(x) = (2 - x)^4$ **[4]**

31 Find the range of values of x for which the function $f(x) = \ln(x^2 + 1)$ is concave.
Show your working. **[5]**

32 A sector of a circle has radius r and angle θ as shown.

Given that the arc length is 6 cm.



a Show that the area of the triangle, A, can be expressed as

$A = \frac{18}{\theta^2}\sin\theta$ **[5]**

b Show that the stationary point occurs when $\tan\theta = \frac{\theta}{2}$ **[4]**

33 $f(x) = x - \ln(2x - 3)$, $x > \frac{3}{2}$

The tangent to the curve $y = f(x)$ at $x = 3$ intersects the x-axis at point P

Find the x-coordinate of P, giving your answer in the form $a + \ln b$. Show your working. **[7]**

34 Given that $y = \text{cosec}\,3x$

a Show that $\frac{dy}{dx} = A\cot 3x\,\text{cosec}\,3x$, where A is a constant to be found, **[4]**

b Find $\frac{d^2y}{dx^2}$ **[4]**

35 Given that $y = \arctan x$

a Find $\dfrac{dy}{dx}$ **[4]**

b Find an equation of the tangent to $y = \arctan x$ at the point where $x = 1$ **[3]**

The tangent intersects the x-axis at point A and the y-axis at point B

c Show that the area of triangle OAB is $\dfrac{1}{16}(\pi^2 - 4\pi + 4)$ **[5]**

36 A cube has side length x

The volume of the cube is increasing at a rate of $12\ \text{cm}^3\text{s}^{-1}$

Find the rate at which x is increasing when the volume is $216\ \text{cm}^3$ **[6]**

37 The volume of a spherical balloon, $V\ \text{cm}^3$, is increasing at a constant rate of $6\ \text{cm}^3\,\text{s}^{-1}$

Find the rate at which the radius of the sphere is increasing when the volume is $36\pi\ \text{cm}^3$

Leave your answer in exact form. $\left[V = \dfrac{4}{3}\pi r^3\right]$ **[5]**

38 Prove that the derivative of $\arcsin 2x$ is $\dfrac{2}{\sqrt{1-4x^2}}$ **[6]**

39 $xy^2 + 2y = 3x^2$

a Find an expression in terms of x and y for $\dfrac{dy}{dx}$ **[4]**

b Calculate the possible rates of change of y with respect to x when $y = 1$ **[5]**

40 Use implicit differentiation to prove that the derivative of a^x is $a^x \ln a$ **[4]**

41 Given that $x = \dfrac{2}{3-t}$, $y = \dfrac{t^2}{3-t}$, $t \neq 3$,

a Show that $\dfrac{dy}{dx} = \dfrac{6t - t^2}{2}$ **[5]**

b Find a Cartesian equation in the form $y = f(x)$. Simplify your answer. **[3]**

42 A curve C is defined by the parametric equations $x = \sec(\theta - 4)$, $y = \tan(\theta - 4)$

a Show that $\dfrac{dy}{dx} = \operatorname{cosec}(\theta - 4)$ **[5]**

b Find a Cartesian equation of C **[2]**

c Show that the equation of the tangent to C at the point where $x = 3$ and y is positive is given by $3x - 2y\sqrt{2} = 1$ **[6]**