1) $x^2 + y^2 - 2x + 6y = 26$

$x^2 - 2x + y^2 + 6y = 26$

$(x-1)^2 - 1 + (y+3)^2 - 9 = 26$

$(x-1)^2 + (y+3)^2 = 36$

i centre: $(1, -3)$

ii radius: 6

2) $(x-a)^2 + (y-b)^2 = r^2$

$(x-2)^2 + (y-5)^2 = r^2$ $(4, 9)$

$(4-2)^2 + (9-5)^2 = r^2$

$20 = r^2$

~~$r = \sqrt{20}$~~

$(x-2)^2 + (y-5)^2 = 20$

3) $(x-a)^2 + (y-b)^2 = r^2$

$(x+2)^2 + (y-3)^2 = r^2$ $\quad (1, 8)$

$(1+2)^2 + (8-3)^2 = r^2$

$34 = r^2$

$(x+2)^2 + (y-3)^2 = 34$

b/ $(3+2)^2 + (6-3)^2 = 34$

$34 = 34$

c/ centre $(-2, 3)$ $\quad$ point $(3, 6)$

$x_1\ y_1$ $\quad$ $x_2\ y_2$

gradient of radius $= \frac{6-3}{3--2}$

$= \frac{3}{5}$

gradient of tangent $= -5/3$

$y = -\frac{5}{3}x + c \qquad (3, 6)$

$6 = -\frac{5}{3}(3) + c$

$6 = -5 + c$

$c = 11$

$y = -\frac{5}{3}x + 11$

$3y = -5x + 33$

$5x + 3y - 33 = 0$

4a) $(x-2)^2 + (y-5)^2 = 49$

b) $y = (3x - 1)$

$$(x-2)^2 + (3x-1-5)^2 = 49$$

$$(x-2)^2 + (3x-6)^2 = 49$$

$$(x-2)(x-2) + (3x-6)(3x-6) = 49$$

$$x^2 - 2x - 2x + 4 + 9x^2 - 18x - 18x + 36 = 49$$

$$10x^2 - 40x + 40 = 49$$

$$10x^2 - 40x - 9 = 0$$

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(10)(-9)}}{2(10)}$$

$$x = \frac{20 + 7\sqrt{10}}{10} \qquad x = \frac{20 - 7\sqrt{10}}{10}$$

$$y = 3\left(\frac{20+7\sqrt{10}}{10}\right) - 1 \qquad y = 3\left(\frac{20-7\sqrt{10}}{10}\right) - 1$$

$$= \frac{50 + 21\sqrt{10}}{10} \qquad = \frac{50 - 21\sqrt{10}}{10}$$

$$\left(\frac{20+7\sqrt{10}}{10}, \frac{50+21\sqrt{10}}{10}\right) \text{ and } \left(\frac{20-7\sqrt{10}}{10}, \frac{50-21\sqrt{10}}{10}\right)$$

5) $x^2 + y^2 + 8x - 4y + k = 0$ $(1, 5)$

$(1)^2 + (5)^2 + 8(1) - 4(5) + k = 0$

$14 + k = 0$

$k = -14$

b) $x^2 + y^2 + 8x - 4y - 14 = 0$

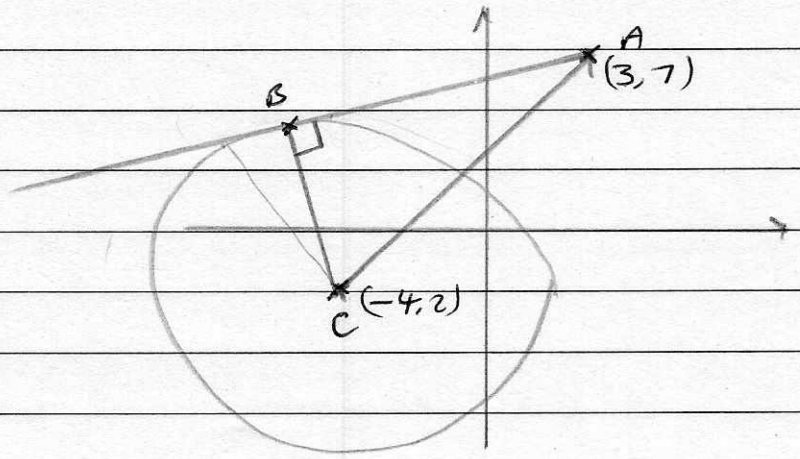
$(x+4)^2 - 16 + (y-2)^2 - 4 - 14 = 0$

$(x+4)^2 + (y-2)^2 = 34$

centre: $(-4, 2)$

radius: $\sqrt{34}$

c)



BC is a radius: $\sqrt{34}$

$AC = \sqrt{7^2 + 5^2} = \sqrt{74}$

$AB^2 + BC^2 = AC^2$

$AB^2 + 34 = 74$

$AB^2 = 40$

$AB = \sqrt{40}$

$= 2\sqrt{10}$

6a) D(−3, 2) E(4, −1) F(1, −8)

$DE^2 = 7^2 + 3^2$ $DF^2 = 4^2 + 10^2$ $EF^2 = 3^2 + 7^2$

$DE = \sqrt{58}$ $DF = \sqrt{116}$ $EF = \sqrt{58}$

DEF is a right angled triangle as $DE^2 + EF^2 = DF^2$

$(58 + 58 = 116)$

DF is the hypotonuse $\therefore$ $\underline{DEF = 90^\circ}$

b) DEF is 90° $\therefore$ DF is a diameter.

$\left(\frac{-3+1}{2}, \frac{2-8}{2}\right)$

$\underline{(-1, -3)}$

c) $(x+1)^2 + (y+3)^2 = r^2$

$r = \frac{\sqrt{116}}{2} = \sqrt{29}$

$r^2 = 29$

$\underline{(x+1)^2 + (y+3)^2 = 29}$

7a) $x^2 + y^2 - 6x + 2y = 6$

$(x-3)^2 - 9 + (y+1)^2 - 1 = 6$

$(x-3)^2 + (y+1)^2 = 16$

Centre: $(3, -1)$

radius: 6

b) crosses y when $x = 0$

$(0-3)^2 + (y+1)^2 = 16$

$9 + (y+1)^2 = 16$

$(y+1)^2 = 7$

$y+1 = \pm\sqrt{7}$

$y = -1 \pm \sqrt{7}$

$(0, -1+\sqrt{7})$ $(0, -1-\sqrt{7})$

8/ $(-3, 5)$ $(13, -4)$

centre = midpoint of diameter

$$\left(\frac{-3+13}{2}, \frac{5-4}{2}\right)$$

$$\left(5, \frac{1}{2}\right)$$

$$(x-5)^2 + \left(y-\frac{1}{2}\right)^2 = r^2$$

$$\text{diameter length} = \sqrt{16^2 + 9^2}$$

$$= \sqrt{337}$$

$$\text{radius} = \frac{\sqrt{337}}{2}$$

$$r^2 = \frac{337}{4}$$

$$(x-5)^2 + \left(y-\frac{1}{2}\right)^2 = \frac{337}{4}$$

9a/ $(x-1)^2 + (y-5)^2 = r^2$ $(-4, 3)$

$(-5)^2 + (-2)^2 = r^2$

$29 = r^2$

$\underline{(x-1)^2 + (y-5)^2 = 29}$

b/ gradient of radius $= \frac{3-5}{-4-1}$

$= \frac{-2}{-5}$

$= \frac{2}{5}$

gradient of tangent $= -\frac{5}{2}$

$y = -\frac{5}{2}x + c$ $(-4, 3)$

$3 = -\frac{5}{2}(-4) + c$

$3 = 10 + c$

$\underline{c = -7}$

$y = -\frac{5}{2}x - 7$

$2y = -5x - 14$

$\underline{5x + 2y + 14 = 0}$

10) gradient of tangent = 2

gradient of radius = $-\frac{1}{2}$

$$-\frac{1}{2} = \frac{7-k}{3-5}$$

$$-\frac{1}{2} = \frac{7-k}{-2}$$

$$-1(-2) = 2(7-k)$$

$$2 = 14 - 2k$$

$$2k = 12$$

$$\underline{k = 6}$$

$$(x-5)^2 + (y-6)^2 = r^2 \qquad (3,7)$$

$$(3-5)^2 + (7-6)^2 = r^2$$

$$5 = r^2$$

$$\underline{(x-5)^2 + (y-6)^2 = 5}$$